

**ON A TYPE OF QUARTER-SYMMETRIC NON-METRIC  
 $\phi$ -CONNECTION ON A KENMOTSU MANIFOLD**

**(COMMUNICATED BY PROFESSOR U. C. DE)**

AJIT BARMAN

ABSTRACT. The object of the present paper is to study a quarter-symmetric non-metric  $\phi$ -connection on a Kenmotsu manifold.

1. INTRODUCTION

The product of an almost contact manifold  $M$  and the real line  $R$  carries a natural almost complex structure. However if one takes  $M$  to be an almost contact metric manifold and suppose that the product metric  $G$  on  $M \times R$  is Kaehlerian, then the structure on  $M$  is cosymplectic [8] and not Sasakian. On the other hand Oubina [13] pointed out that if the conformally related metric  $e^{2t}G$ ,  $t$  being the coordinate on  $R$ , is Kaehlerian, then  $M$  is Sasakian and conversely.

In [17], S. Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold  $M$ , the sectional curvature of plane sections containing  $\xi$  is a constant, say  $c$ . If  $c > 0$ ,  $M$  is a homogeneous Sasakian manifold of constant sectional curvature. If  $c = 0$ ,  $M$  is the product of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature. If  $c < 0$ ,  $M$  is a warped product space  $R \times_f C^n$ . In 1971, Kenmotsu studied a class of contact Riemannian manifolds satisfying some special conditions [10]. We call it Kenmotsu manifold. Kenmotsu manifolds have been studied by J.B. Jun, U.C. De and G. Pathak [9], C. Özgür and U.C. De [14], U.C. De and G. Pathak [4], A. Yıldız, U.C. De and B.E. Acet [19] and others.

In 1975, S. Golab [7] defined and studied quarter-symmetric connection in differentiable manifolds with affine connections. A linear connection  $\nabla$  on an  $n$ -dimensional Riemannian manifold  $(M^n, g)$  is called a quarter-symmetric connection [7] if its

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torsion tensor  $T$  satisfies

$$T(X, Y) = \eta(Y)\phi X - \eta(X)\phi Y, \quad (1.1)$$

where  $\eta$  is a 1-form and  $\phi$  is a (1,1) tensor field.

In particular, if  $\phi X = X$ , then the quarter-symmetric connection reduces to the semi-symmetric connection [6]. Thus the notion of the quarter-symmetric connection generalizes the notion of the semi-symmetric connection.

If moreover, a quarter-symmetric connection  $\nabla$  satisfies the condition

$$(\nabla_X g)(Y, Z) \neq 0, \quad (1.2)$$

then  $\nabla$  is said to be a quarter-symmetric non-metric connection.

The quarter-symmetric non-metric connection is said to be a quarter-symmetric non-metric  $\phi$ -connection if satisfies the condition

$$(\nabla_X \phi)(Y) = 0, \quad (1.3)$$

for all  $X, Y, Z \in \chi(M^n)$ .

After S. Golab [7] and S.C.Rastogi ([15], [16]) continued the systematic study of quarter-symmetric metric connection by R.S.Mishra and S.N.Pandey [11], K.Yano and T. Imai [18], S. Mukhopadhyay, A. K. Roy and B. Barua [12], U.C.De and S.C. Biswas [3], U.C. De and G. Pathak [4], J.B. Jun, U.C. De and G. Pathak [9], U.C. De, C. Özgür and S. Sular [5] and others.

A Riemannian manifold is said to be semisymmetric if its curvature tensor  $K$  satisfies the condition

$$K(X, Y).K = 0,$$

where  $K(X, Y)$  denotes the curvature operator and Ricci-semisymmetric if

$$K(X, Y).\tilde{S} = 0,$$

where  $\tilde{S}$  denotes the Ricci tensor of the manifold.

In this paper we study Kenmotsu manifolds with respect to the quarter-symmetric non-metric  $\phi$ -connection. The paper is organized as follows: After introduction in section 2, we give a brief account of the Kenmotsu manifolds. In section 3, we define a quarter-symmetric non-metric  $\phi$ -connection on a Kenmotsu manifold and we establish the relation between the curvature tensors with respect to the quarter-symmetric non-metric  $\phi$ -connection and the Levi-Civita connection. Section 4, deals with  $R.R = 0$  in a Kenmotsu manifold with respect to the quarter-symmetric non-metric  $\phi$ -connection. In section 5, we investigate  $R.S = 0$  in a Kenmotsu

manifold with respect to the quarter-symmetric non-metric  $\phi$ -connection and we prove that the manifold is Ricci-semisymmetric with respect to the Levi-Civita connection. Finally, we study  $S.R = 0$  in a Kenmotsu manifold with respect to the quarter-symmetric non-metric  $\phi$ -connection, where  $R$  and  $S$  denotes the curvature tensor and the Ricci tensor of the Kenmotsu manifold with respect to the quarter-symmetric non-metric  $\phi$ -connection respectively.

## 2. KENMOTSU MANIFOLDS

Let  $M$  be an  $(2n+1)$ -dimensional almost contact metric manifold with an almost contact metric structure  $(\phi, \xi, \eta, g)$  consisting of a  $(1, 1)$  tensor field  $\phi$ , a vector field  $\xi$ , a 1-form  $\eta$  and a Riemannian metric  $g$  on  $M$  satisfying [2]

$$\phi^2(X) = -X + \eta(X)\xi, \quad g(X, \xi) = \eta(X), \quad (2.1)$$

$$\eta(\xi) = 1, \quad \phi(\xi) = 0, \quad \eta(\phi(X)) = 0, \quad (2.2)$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad (2.3)$$

$$g(X, \phi Y) = -g(\phi X, Y). \quad (2.4)$$

for all vector fields  $X, Y$  on  $M$ . If an almost contact metric manifold satisfies

$$(D_X \phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X, \quad (2.5)$$

then  $M$  is called a Kenmotsu manifold [10]. From the above relations, it follows that

$$D_X \xi = X - \eta(X)\xi, \quad (2.6)$$

$$(D_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y). \quad (2.7)$$

Moreover the curvature tensor  $K$  and the Ricci tensor  $\tilde{S}$  and the Ricci operator  $\tilde{Q}$  of the Kenmotsu manifold with respect to the Levi-Civita connection satisfies

$$K(X, Y)\xi = \eta(X)Y - \eta(Y)X, \quad (2.8)$$

$$K(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \quad (2.9)$$

$$K(X, \xi)Y = g(X, Y)\xi - \eta(Y)X, \quad (2.10)$$

$$\tilde{S}(\phi X, \phi Y) = \tilde{S}(X, Y) + 2n\eta(X)\eta(Y), \quad (2.11)$$

$$\tilde{S}(X, Y) = g(\tilde{Q}X, Y) = -2ng(X, Y). \quad (2.12)$$

$$\tilde{Q}X = -2nX. \quad (2.13)$$

$$\tilde{S}(X, \xi) = -2n\eta(X). \quad (2.14)$$

3. CURVATURE TENSOR WITH RESPECT TO THE QUARTER-SYMMETRIC  
NON-METRIC  $\phi$ -CONNECTION

Let  $(M^{2n+1}, g)$  be a Kenmotsu Manifold with the Levi-Civita connection  $D$ . We define a linear connection  $\nabla$  on  $M$  by

$$\nabla_X Y = D_X Y - \eta(X)\phi Y + g(X, Y)\xi - \eta(Y)X - \eta(X)Y. \quad (3.1)$$

Using (3.1), the torsion tensor  $T$  of  $M$  with respect to the connection  $\nabla$  is given by

$$T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y] = \eta(Y)\phi X - \eta(X)\phi Y. \quad (3.2)$$

A linear connection satisfying (3.2) is called a quarter-symmetric connection. Further using (3.1), we have

$$\begin{aligned} (\nabla_X g)(Y, Z) &= Xg(Y, Z) - g(\nabla_X Y, Z) \\ &\quad - g(Y, \nabla_X Z) = 2\eta(X)g(Y, Z) \neq 0. \end{aligned} \quad (3.3)$$

A linear connection  $\nabla$  satisfying (3.2) and (3.3) is called a quarter-symmetric non-metric connection.

Again using (3.1), it follows that

$$(\nabla_X \phi)(Y) = \nabla_X \phi Y - \phi(\nabla_X Y) = 0, \quad (3.4)$$

A linear connection  $\nabla$  define by (3.1) satisfying (3.2), (3.3) and (3.4) is called a quarter-symmetric non-metric  $\phi$ -connection.

Conversely, we show that a linear connection  $\nabla$  defined on  $M$  satisfying (3.2), (3.3) and (3.4) is given by (3.1).

Let  $H$  be a tensor field of type  $(1, 2)$  and

$$\nabla_X Y = D_X Y + H(X, Y). \quad (3.5)$$

Then we have

$$T(X, Y) = H(X, Y) - H(Y, X). \quad (3.6)$$

Further using (3.5), it follows that

$$\begin{aligned} (\nabla_X g)(Y, Z) &= \nabla_X g(Y, Z) - g(\nabla_X Y, Z) - g(Y, \nabla_X Z) = -g(H(X, Y), Z) \\ &\quad - g(Y, H(X, Z)). \end{aligned} \quad (3.7)$$

From (3.3) and (3.7), we obtain

$$g(H(X, Y), Z) + g(Y, H(X, Z)) = -2\eta(X)g(Y, Z). \quad (3.8)$$

Also using (3.8) and (3.6), we get

$$g(T(X, Y), Z) + g(T(Z, X), Y) + g(T(Z, Y), X) = 2g(H(X, Y), Z) + 2\eta(X)g(Y, Z) \\ + 2\eta(Y)g(X, Z) - 2\eta(Z)g(X, Y). \quad (3.9)$$

Hence,

$$g(H(X, Y), Z) = \frac{1}{2}[g(T(X, Y), Z) + g(T(Z, X), Y) + g(T(Z, Y), X)] - \eta(X)g(Y, Z) \\ - \eta(Y)g(X, Z) + \eta(Z)g(X, Y). \quad (3.10)$$

Let  $T'$  be a tensor field of type (1, 2) given by

$$g(T'(X, Y), Z) = g(T(Z, X), Y). \quad (3.11)$$

Then

$$T'(X, Y) = g(X, \phi Y)\xi - \eta(X)\phi Y. \quad (3.12)$$

From (3.10) we have by using (3.11) and (3.12)

$$g(H(X, Y), Z) = \frac{1}{2}[g(T(X, Y), Z) + g(T'(X, Y), Y) + g(T'(Y, X), X)] - \eta(X)g(Y, Z) \\ - \eta(Y)g(X, Z) + \eta(Z)g(X, Y) = -\eta(X)g(\phi Y, Z) - \eta(X)g(Y, Z) - \eta(Y)g(X, Z) \\ + \eta(Z)g(X, Y). \quad (3.13)$$

Hence,

$$H(X, Y) = -\eta(X)\phi Y - \eta(X)Y - \eta(Y)X + g(X, Y)\xi. \quad (3.14)$$

From (3.5) and (3.14), it follows that

$$\nabla_X Y = D_X Y - \eta(X)\phi Y + g(X, Y)\xi - \eta(Y)X - \eta(X)Y.$$

Analogous to the definitions of the curvature tensor of  $M$  with respect to the Levi-Civita connection  $D$ , we define the curvature tensor of  $M$  with respect to the quarter-symmetric non-metric  $\phi$ -connection  $\nabla$  by

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z, \quad (3.15)$$

where  $R$  be the curvature tensor with respect to the quarter-symmetric non-metric  $\phi$ -connection  $\nabla$ .

From (3.1) and (3.15), we obtain

$$\begin{aligned}
R(X, Y)Z &= K(X, Y)Z + \eta(X)(D_Y\phi)(Z) - \eta(Y)(D_X\phi)(Z) \\
&\quad + \eta(X)\eta(Z)\phi Y - \eta(Y)\eta(Z)\phi X + \eta(Y)\eta(Z)X \\
&\quad - \eta(X)\eta(Z)Y - \eta(Y)g(X, \phi Z)\xi + \eta(X)g(Y, \phi Z)\xi \\
&\quad + g(Y, Z)D_X\xi - g(X, Z)D_Y\xi + g(Y, Z)\eta(X)\xi \\
&\quad - g(X, Z)\eta(Y)\xi - g(Y, Z)X + g(X, Z)Y \\
&\quad - (D_X\eta)(Y)Z + (D_Y\eta)(X)Z + (D_Y\eta)(X) \\
&\quad - (D_X\eta)(Y) - (D_X\eta)(Y)\phi Z + (D_Y\eta)(X)\phi Z. \tag{3.16}
\end{aligned}$$

Using (2.5), (2.6), (2.7) in (3.16), we have

$$\begin{aligned}
R(X, Y)Z &= K(X, Y)Z + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y + g(Y, Z)X \\
&\quad - g(X, Z)Y. \tag{3.17}
\end{aligned}$$

From (3.17), it follows that the curvature tensor  $R$  satisfies

$$R(X, Y)Z + R(Y, Z)X + R(Z, X)Y = 0, \tag{3.18}$$

and

$$R(X, Y)Z = -R(Y, X)Z, \tag{3.19}$$

which implies that  $R$  satisfies the first Bianchi identity and skew-symmetric with respect to the first two variables with respect to the quarter-symmetric non-metric  $\phi$ -connection  $\nabla$ .

Taking the inner product of (3.17) with  $W$ , it follows that

$$\begin{aligned}
\tilde{R}(X, Y, Z, W) &= \tilde{K}(X, Y, Z, W) + \eta(Y)\eta(Z)g(X, W) - \eta(X)\eta(Z)g(Y, W) \\
&\quad + g(Y, Z)g(X, W) - g(X, Z)g(Y, W), \tag{3.20}
\end{aligned}$$

where  $\tilde{R}(X, Y, Z, W) = g(R(X, Y)Z, W)$  and  $\tilde{K}(X, Y, Z, W) = g(K(X, Y)Z, W)$ .

From (3.20) yields,

$$\tilde{R}(X, Y, Z, W) = -\tilde{R}(Y, X, Z, W), \tag{3.21}$$

and

$$\tilde{R}(X, Y, Z, W) = -\tilde{R}(X, Y, W, Z). \tag{3.22}$$

Contracting (3.20) over  $X$  and  $W$ , we obtain

$$S(Y, Z) = \tilde{S}(Y, Z) + 2n\eta(Y)\eta(Z) + 2ng(Y, Z), \tag{3.23}$$

where  $S$  be the Ricci tensor with respect to the quarter-symmetric non-metric  $\phi$ -connection  $\nabla$ .

From (3.23), we have

$$S(Y, Z) = S(Z, Y), \tag{3.24}$$

And putting  $Z = \xi$  in (3.23) and using (2.14), we get

$$S(Y, \xi) = 2n\eta(Y). \tag{3.25}$$

Again contracting (3.23) over  $Y$  and  $Z$ , it follows that

$$r = \tilde{r} + 2n(2n + 2), \tag{3.26}$$

where  $r$  and  $\tilde{r}$  are the scalar curvatures with respect to the quarter-symmetric non-metric  $\phi$ -connection  $\nabla$  and the Levi-Civita connection  $D$  respectively.

From the above discussions we can state as follows:

**Theorem 3.1.** *For a Kenmotsu manifold  $M$  with respect to the quarter-symmetric non-metric  $\phi$ -connection  $\nabla$*

- (i) *The curvature tensor  $R$  is given by (3.17),*
- (ii) *The Ricci tensor  $S$  is given by (3.23),*
- (iii)  *$R(X, Y)Z + R(Y, Z)X + R(Z, X)Y = 0,$*
- (iv)  *$R(X, Y)Z = -R(Y, X)Z,$*
- (v)  *$\tilde{R}(X, Y, Z, W) + \tilde{R}(Y, X, Z, W) = 0,$*
- (vi)  *$\tilde{R}(X, Y, Z, W) + \tilde{R}(X, Y, W, Z) = 0,$*
- (vii)  *$S(Y, \xi) = 2n\eta(Y),$*
- (viii)  *$r = \tilde{r} + 2n(2n + 2),$*
- (ix) *The Ricci tensor  $S$  is symmetric.*

4. KENMOTSU MANIFOLDS WITH RESPECT TO THE QUARTER-SYMMETRIC NON-METRIC  $\phi$ -CONNECTION  $\nabla$  SATISFYING  $R.R = 0$

**Definition 4.1.** *A Kenmotsu manifold  $M^{2n+1}, (n > 1)$  is said to be an Einstein manifold if its Ricci tensor  $\tilde{S}$  of the Levi-Civita connection is of the form*

$$\tilde{S}(X, Y) = ag(X, Y), \tag{4.1}$$

where  $a$  is a constant on the manifold .

In this section we suppose that the manifold under consideration is semisymmetric with respect to the quarter-symmetric non-metric  $\phi$ -connection  $M^{2n+1}$ , that is,

$$(R(X, Y).R)(U, V)W = 0$$

Then we have

$$\begin{aligned} (R(X, Y))R(U, V)W - R(R(X, Y)U, V)W - R(U, R(X, Y)V)W \\ - R(U, V)R(X, Y)W = 0. \end{aligned} \tag{4.2}$$

Putting  $X = \xi$  in (4.2), it follows that

$$\begin{aligned} R(\xi, Y)R(U, V)W - R(R(\xi, Y)U, V)W - R(U, R(\xi, Y)V)W \\ - R(U, V)R(\xi, Y)W = 0. \end{aligned} \tag{4.3}$$

Putting  $U = \xi$  in (4.3), we obtain

$$\begin{aligned} R(\xi, Y)R(\xi, V)W - R(R(\xi, Y)\xi, V)W - R(\xi, R(\xi, Y)V)W \\ - R(\xi, V)R(\xi, Y)W = 0. \end{aligned} \quad (4.4)$$

Using (2.8), (2.9), (2.10), (2.1), (2.2) and (3.17) in (4.4), we have

$$K(Y, V)W = g(Y, W)V - g(V, W)Y. \quad (4.5)$$

From (4.5), it follows that the manifold is a manifold of constant curvature  $-1$ , that is, the manifold under consideration is locally isometric to the hyperbolic space  $H_n(-1)$ .

Conversly if the manifold is a manifold of constant curvature  $-1$ , then it is semisymmetric ( $K.K=0$ ).

Hence we can state the following:

**Theorem 4.1.** *If a Kenmotsu manifold is semisymmetric with respect to the quarter-symmetric non-metric  $\phi$ -connection, then the manifold is semisymmetric with respect to the Levi-Civita connection.*

#### 5. KENMOTSU MANIFOLDS WITH RESPECT TO THE QUARTER-SYMMETRIC NON-METRIC $\phi$ -CONNECTION $\nabla$ SATISFYING $R.S = 0$

In this section we suppose that the manifold under consideration is Ricci-semisymmetric with respect to the quarter-symmetric non-metric  $\phi$ -connection  $M^{2n+1}$ , that is,

$$(R(X, Y).S)(U, V) = 0$$

Then we have

$$S(R(X, Y)U, V) + S(U, R(X, Y)V) = 0. \quad (5.1)$$

Putting  $X = \xi$  in (5.1), it follows that

$$S(R(\xi, Y)U, V) + S(U, R(\xi, Y)V) = 0. \quad (5.2)$$

Using (2.9), (2.14), (2.1), (2.2) and (3.17) in (5.2), we obtain

$$\eta(U)S(Y, V) + \eta(V)S(Y, U) = -4n\eta(U)\eta(V)\eta(Y). \quad (5.3)$$

Putting  $U = \xi$  in (5.3) and using (2.1) and (2.2), we get

$$\tilde{S}(Y, V) = -2ng(Y, V). \quad (5.4)$$

Therefore ,  $\tilde{S}(Y, V) = ag(Y, V)$ ,

where  $a = -2n$ .

This result shows that the manifold is an Einstein manifold.

Conversly if the manifold is an Einstein manifold, then the manifold is Ricci-semisymmetric ( $K.\tilde{S} = 0$ ).

Therefore, we can state the following:

**Theorem 5.1.** *If a Kenmotsu manifold is Ricci-semisymmetric with respect to the quarter-symmetric non-metric  $\phi$ -connection, then the manifold is Ricci-semisymmetric with respect to the Levi-Civita connection.*

#### 6. KENMOTSU MANIFOLDS WITH RESPECT TO THE QUARTER-SYMMETRIC NON-METRIC $\phi$ -CONNECTION $\nabla$ SATISFYING $S.R = 0$

In this section we suppose that the manifold under consideration is satisfied  $S.R = 0$  with respect to the quarter-symmetric non-metric  $\phi$ -connection  $M^{2n+1}$ , that is,

$$(S(X, Y).R)(U, V)W = 0. \quad (6.1)$$

This implies

$$(X \wedge_S Y)R(U, V)W + R((X \wedge_S Y)U, V)W + R(U, (X \wedge_S Y)V)W + R(U, V)(X \wedge_S Y)W = 0, \quad (6.2)$$

where the endomorphism  $X \wedge_S Y$  is defined by

$$(X \wedge_S Y)W = S(Y, W)X - S(X, W)Y. \quad (6.3)$$

Using the above we obtain

$$\begin{aligned} & S(Y, R(U, V)W)X - S(X, R(U, V)W)Y + S(Y, U)R(X, V)W \\ & - S(X, U)R(Y, V)W + S(Y, V)R(U, X)W - S(X, V)R(U, Y)W \\ & + S(Y, W)R(U, V)X - S(X, W)R(U, V)Y = 0. \end{aligned} \quad (6.4)$$

Putting  $Y = \xi$  in (6.4) and using (3.17), (3.25) and (3.23), we get

$$\begin{aligned} & 2n\eta(R(U, V)W)X - S(X, R(U, V)W)\xi + 2n\eta(U)R(X, V)W \\ & - S(X, U)[K(\xi, V)W + \eta(V)\eta(W)\xi - \eta(W)V + g(V, W)\xi \\ & - \eta(W)V] + 2n\eta(V)R(U, X)W - S(X, V)[K(U, \xi)W + \eta(W)U \\ & - \eta(U)\eta(W)\xi + \eta(W)U - g(U, W)\xi] + 2n\eta(W)R(U, V)X \\ & - S(X, W)[K(U, V)\xi + 2\eta(V)U - 2\eta(U)V] = 0. \end{aligned} \quad (6.5)$$

Putting  $U = \xi$  in (6.5) and using (3.17), (3.25), (2.8), (2.9), (2.10), (2.14) and (3.23), we have

$$\begin{aligned} \eta(W)S(X, V)\xi + 2nK(X, V)W - 2n\eta(X)\eta(W)V \\ + 2ng(V, W)X - 2ng(X, W)V \\ - \eta(V)S(X, W)\xi + S(X, W)V = 0. \end{aligned} \quad (6.6)$$

Contracting  $X$  in (6.6) and using (2.1), (2.2) and (2.14), we obtain

$$\tilde{S}(V, W) = -2ng(V, W). \quad (6.7)$$

Therefore,

$$\tilde{S}(Y, W) = ag(Y, W),$$

where  $a = -2n$

This result shows that the manifold is an Einstein manifold.

Hence we can state the following theorem:

**Theorem 6.1.** *If a Kenmotsu manifold with respect to the quarter-symmetric non-metric  $\phi$ -connection satisfies  $S.R = 0$ , then the manifold is an Einstein manifold.*

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AJIT BARMAN  
DEPARTMENT OF MATHEMATICS,  
KABI-NAZRUL MAHAVIDYALAYA,  
P.O.:-SONAMURA-799181,  
P.S.-SONAMURA,  
DIST- SEPAHIJALA ,  
TRIPURA, INDIA.

*E-mail address:* [ajitbarmanaw@yahoo.in](mailto:ajitbarmanaw@yahoo.in)