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SOME STRONG CONVERGENCE RESULTS FOR MANN AND ISHIKAWA ITERATIVE PROCESSES IN BANACH SPACES

(DEDICATED IN OCCASION OF THE 70-YEARS OF PROFESSOR HARI M. SRIVASTAVA)

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Abstract. In this paper, we establish some strong convergence results for Mann and Ishikawa iterative processes in a Banach space setting by employing some general contractive conditions as well as weakening further the conditions on the parameter sequence $\{\alpha_n\} \subset [0,1]$. In addition, in some of our results, we introduce some innovative ideas which make our results distinct from some previous ones. In particular, our results generalize, extend and improve those of [V. Berinde; On the convergence of Mann iteration for a class of quasi-contractive operators, Preprint, North University of Baia Mare (2003)] and [V. Berinde; On the Convergence of the Ishikawa Iteration in the Class of Quasi-contractive Operators, Acta Math. Univ. Comenianae Vol. LXXIII (1) (2004), 119-126] as well as some other analogous results in the literature.

1. Introduction

Let (E,d) be a complete metric space and $T : E \to E$ a selfmap of E. Suppose that $F_T = \{p \in E \mid Tp = p\}$ is the set of fixed points of T.
There are several iteration processes in the literature for which the fixed points of operators have been approximated over the years by various authors. In a complete metric space, the Picard iteration process $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = Tx_n,\ n = 0, 1, \cdots, \tag{1}$$

has been employed to approximate the fixed points of mappings satisfying the inequality relation

$$d(Tx,Ty) \le \alpha d(x,y),\ \forall\ x,\ y \in E \text{ and } \alpha \in [0,1). \tag{2}$$

Condition (2) is called the *Banach's contraction condition.* Also, condition (2) is significant in the celebrated Banach's fixed point theorem [2].

In the Banach space setting, we have the following iterative processes generalizing iteration (1):

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For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n,\ n = 0, 1, \cdots, \tag{3}$$

where $\{\alpha_n\}_{n=0}^{\infty} \subset [0,1]$, is called the *Mann iterative* process (see Mann [15]).
For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\left.\begin{array}{l} x_{n+1} = (1-\alpha_n)x_n + \alpha_n T z_n \\ z_n = (1-\beta_n)x_n + \beta_n T x_n \end{array}\right\} \ n = 0, 1, \cdots, \tag{4}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are sequences in $[0,1]$, is called the *Ishikawa iterative* process (see Ishikawa [11]).

Zamfirescu [21] established a nice generalization of the Banach's fixed point theorem by employing the following contractive condition: For a mapping $T : E \to E$, there exist real numbers $\alpha,\ \beta, \gamma$ satisfying $0 \le \alpha < 1,\ 0 \le \beta < \frac{1}{2},\ 0 \le \gamma < \frac{1}{2}$ respectively such that for each $x,\ y \in E$, at least one of the following is true:

$$\left.\begin{array}{l} (z_1)\ d(Tx,Ty) \le \alpha d(x,y) \\ (z_2)\ d(Tx,Ty) \le \beta\left[d(x,Tx) + d(y,Ty)\right] \\ (z_3)\ d(Tx,Ty) \le \gamma\left[d(x,Ty) + d(y,Tx)\right]. \end{array}\right\} \tag{5}$$

The mapping $T : E \to E$ satisfying (5) is called the *Zamfirescu contraction.* Any mapping satisfying condition (z_2) of (5) is called a *Kannan mapping,* while the mapping satisfying condition (z_3) is called *Chatterjea operator.* For more on conditions (z_2) and (z_3), we refer to Kannan [12] and Chatterjea [8] respectively. It has been shown in Berinde [4] that the contractive condition (5) implies

$$d(Tx,Ty) \le 2\delta d(x,Tx) + \delta d(x,y),\ \forall\ x,\ y \in E, \tag{6}$$

where $\delta = \max\left\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\right\},\ 0 \le \delta < 1.$
Consequently, the author [3, 4] used (6) to prove strong convergence results in Banach space setting for Mann and Ishikawa iterations.

More recently, Berinde [7] established several results including the following generalization of Banach's fixed point theorem:

Theorem 1.1. *Let (E,d) be a complete metric space and $T\colon E \to E$ be a mapping for which there exists $\alpha \in [0,1)$ and some $L \ge 0$ such that for all $x,\ y \in E$,*

$$d(Tx,Ty) \le \alpha M_1(x,y) + Lm(x,y), \tag{7}$$

where $M_1(x,y) = \max\{\mathrm{d(x,y), d(x,Tx), d(y,Ty)}, \frac{1}{2}[\mathrm{d(x,Ty) + d(y,Tx)}]\}$,
and $m(x,y) = \min\{\mathrm{d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)}\}$.
Then:
(i) T has a unique fixed point, i.e. Fix$(T) = \{x^*\}$;
(ii) The Picard iteration $\{x_n\}_{n=0}^{\infty}$ given by (1) converges to x^*, for any $x_0 \in E$;
(iii) The error estimate

$$d(x_{n+i-1}, x^*) \le \frac{\alpha^i}{1-\alpha} d(x_n, x_{n-1}),\ n = 0, 1, 2, \cdots;\ i = 1, 2, \cdots,$$

holds.

Remark 1.1: Theorem 1.1 is exactly Theorem 2.4 in Berinde [7].

Motivated by condition (7) of Theorem 1.1, we now state the following contractive condition which shall be used in proving our results: For a mapping $T\colon E \to E$, there exists $\delta \in [0,1)$ and some $L \geq 0$ such that for all $x,\ y \in E$, we have

$$d(Tx,Ty) \leq \delta d(x,y) + Lm(x,y), \tag{8}$$

where

$$\begin{aligned} m(x,y) &= \min\{\mathrm{d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)}, \\ &\tfrac{1}{2}[d(x,Tx)+d(y,Ty)], \tfrac{1}{2}[d(x,Ty)+d(y,Tx)]\}. \end{aligned}$$

Remark 1.2: (i) Condition (8) is independent of (7).
(ii) If in (8), $m(x,y) = d(x,Tx)$, then we obtain the contractive condition of Theorem 2.3 (Berinde [7]).
(iii) Condition (8) reduces to that of Popescu [16] if
$m(x,y) = \min\{\mathrm{d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)}\}$.
(iv) Condition (8) reduces to those of Banach [2], Chatterjea [8], Kannan [12], Zamfirescu [21] and some others by suitable choices of $\delta,\ L$ and $m(x,y)$.
(v) Condition (8) shall be employed in the Banach space setting with
$d(x,y) = ||x-y||,\ \forall\ x,\ y \in E$, since metric is induced by norm.

2. Main Results

Theorem 2.1. *Let $(E,||.||)$ be an arbitrary Banach space, K a closed convex subset of E and $T\colon K \to K$ an operator satisfying (8). For $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ defined by (4) be the Ishikawa iterative process with $\alpha_n,\ \beta_n \in [0,1]$ such that $0 < \alpha \leq \alpha_n,\ \forall\ n$. Then, the Ishikawa iteration $\{x_n\}_{n=0}^{\infty}$ converges strongly to the fixed point of T.*

Proof. We shall first establish that T has a unique fixed point by using condition (8): Suppose not. Then, there exist $x^*,\ y^* \in F_T,\ x^* \neq y^*$ and $||\mathrm{x^*-y^*}|| > 0$. Therefore, we have that

$$\begin{aligned} 0 < ||x^*-y^*|| &= ||Tx^*-Ty^*|| \\ &\leq \delta||x^*-y^*|| + L\min\{||\mathrm{x^*-Tx^*}||, ||\mathrm{y^*-Ty^*}||, ||\mathrm{x^*-Ty^*}||, ||\mathrm{y^*-Tx^*}||, \\ &\tfrac{1}{2}[||x^*-Tx^*||+||y^*-Ty^*||], \tfrac{1}{2}[||x^*-Ty^*||+||y^*-Tx^*||]\} \\ &= \delta||x^*-y^*|| + L\min\{0, ||\mathrm{x^*-y^*}||\} = \delta||\mathrm{x^*-y^*}||, \end{aligned}$$

from which it follows that $||x^*-y^*|| \leq 0,\ \delta \in [0,1)$ (which is a contradiction). Therefore, $||x^*-y^*|| = 0$ i.e. $x^* = y^* = p$, thus proving the uniqueness of the fixed point for T. Hence, $F_T = \{p\}$.

We now prove that $\{x_n\}_{n=0}^{\infty}$ converges strongly to the fixed point p of T using (8): Therefore, we have that

$$\begin{aligned} ||x_{n+1}-p|| &\leq (1-\alpha_n)||x_n-p|| + \alpha_n||Tp-Tz_n|| \\ &\leq (1-\alpha_n)||x_n-p|| + \alpha_n[\delta||p-z_n|| \\ &+L\min\{||\mathrm{p-Tp}||, ||\mathrm{z_n-Tz_n}||, ||\mathrm{p-Tz_n}), ||\mathrm{z_n-Tp}||, \\ &\tfrac{1}{2}[||p-Tp||+||z_n-Tz_n], \tfrac{1}{2}[||p-Tz_n||+||z_n-Tp||]\}] \\ &= (1-\alpha_n)||x_n-p|| + \delta\alpha_n||p-z_n|| \\ &\leq (1-\alpha_n)||x_n-p|| + \delta\alpha_n[(1-\beta_n)||p-x_n|| + \beta_n||Tp-Tx_n||] \\ &\leq [1-\alpha_n(1-\delta) - \delta\alpha_n\beta_n(1-\delta)]||x_n-p|| \\ &= [1-\alpha_n(1-\delta)(1+\delta\beta_n)]||x_n-p||. \ (9) \end{aligned}$$

Now, we have that

$$1-\alpha_n(1-\delta)(1+\delta\beta_n)\leq 1-(1-\delta)^2\alpha_n. \quad (10)$$

Using (10) in (9) as well as the condition on α_n yield

$$\begin{aligned}||x_{n+1}-p|| &\leq [1-(1-\delta)^2\alpha_n]||x_n-p|| \\ &\leq [1-(1-\delta)^2\alpha]||x_n-p|| \\ &\leq [1-(1-\delta)^2\alpha]^2||x_{n-1}-p|| \leq \cdots \leq [1-(1-\delta)^2\alpha]^{n+1}||x_0-p|| \quad (11) \\ &\to 0 \text{ as n} \to \infty, \text{ since } 0<1-(1-\delta)^2\alpha<1.\end{aligned}$$

Hence, we obtain from (11) that $||x_{n+1}-p|| \to 0$ as n $\to \infty$,
that is, $\{x_n\}_{n=0}^{\infty}$ converges strongly to p.

Theorem 2.2. *Let $(E, ||.||)$ be an arbitrary Banach space, K a closed convex subset of E and $T\colon K\to K$ an operator satisfying (8). For $x_0\in K$, let $\{x_n\}_{n=0}^{\infty}$ defined by (3) be the Mann iterative process with $\alpha_n\in[0,1]$ such that $0<\alpha\leq\alpha_n$, $\forall$ n. Then, the Mann iteration $\{x_n\}_{n=0}^{\infty}$ converges strongly to the fixed point of T.*

Proof. The proof of this result is more direct and similar to that of Theorem 2.1.

Remark 2.1: Theorem 2.1 and Theorem 2.2 are generalize, extend and improve a multitude of results. In particular, Theorem 2.1 is a generalization and extension of both Theorem 1 and Theorem 2 of Berinde [4], Theorem 2 and Theorem 3 of Kannan [13], Theorem 3 of Kannan [14], Theorem 4 of Rhoades [17] as well as Theorem 8 of Rhoades [18]. Also, both Theorem 4 of Rhoades [17] and Theorem 8 of Rhoades [18] are Theorem 4.9 and Theorem 5.6 of Berinde [6] respectively. Theorem 2.2 also generalizes and extends the result of Berinde [3, 5], both Theorem 2 and Theorem 3 of Kannan [13], Theorem 3 of Kannan [14] as well as Theorem 4 of Rhoades [17]. Our results also improve the previous results.

Remark 2.2: In the results of Berinde [3, 4, 5], the condition on $\{\alpha_n\}_{n=0}^{\infty}\subset[0,1]$ is $\sum_{n=0}^{\infty}\alpha_n=\infty$. However, this condition has now been removed and replaced by a weaker condition, that is, $0<\alpha\leq\alpha_n$. Thus, our results are improvements over the previous ones in the literature.

Theorem 2.3. *Let E be a set on which two norms $||.||_1$ and $||.||_2$ are defined such that $(E,\ ||.||_1)$ is a Banach space, K a closed convex subset of E and*
$T\colon (K,\ ||.||_1)\to(K,\ ||.||_1)$ a mapping satisfying (8). Suppose that, for arbitrary $x,\ y\in K$, there exists $u\in K$ such that:
(i) $||Ty-y||_2\leq\beta||Tx-x||_2,\ 0<\beta<1$;
(ii) $||u-y||_1\leq\mu||Tx-x||_2,\ \mu>0$.
For $x_0\in K$, let $\{x_n\}_{n=0}^{\infty}$ be the Mann iteration defined by (3) with $\alpha_n\in[0,1]$. Then, the Mann iteration $\{x_n\}_{n=0}^{\infty}$ converges strongly to the fixed point of T.

Proof.

The uniqueness of the fixed point of T has been established in Theorem 2.1 by using condition (8). By (8), we have that

$$\begin{aligned}||x_{n+1}-p||_1 &\leq ||(1-\alpha_n)x_n+\alpha_nTx_n-p||_1 \\ &\leq (1-\alpha_n)||x_n-p||_1+\alpha_n||Tp-Tx_n||_1 \\ &\leq (1-\alpha_n)||x_n-p||_1+\delta\alpha_n||p-x_n||_1 \\ &= [1-(1-\delta)\alpha_n]||x_n-p||_1. \ (12)\end{aligned}$$

Using hypothesis (i), we have that

$$||Tx_{n+1} - x_{n+1}||_2 \leq \beta||Tx_n - x_n||_2 \leq \cdots \leq \beta^{n+1}||Tx_0 - x_0||_2. \quad (13)$$

By (13) and hypothesis (ii), we obtain

$$||p - x_n||_1 \leq \mu\beta^{n-1}||Tx_0 - x_0||_2. \quad (14)$$

Using (14) in (12) yields

$$||x_{n+1} - p||_1 \leq [1 - (1-\delta)\alpha_n]\mu\beta^{n-1}||Tx_0 - x_0||_2 \to \ 0 \text{ as n} \to \infty,$$

from which it follows again that $||x_{n+1} - p|| \to \ 0$ as n $\to \infty$,
that is, $\{x_n\}_{n=0}^{\infty}$ converges strongly to p.

Theorem 2.4. *Let E be a set on which two norms $||.||_1$ and $||.||_2$ are defined such that $(E,\ ||.||_1)$ is a Banach space, K a closed convex subset of E and $T\colon (K,\ ||.||_1) \to (K,\ ||.||_1)$ a mapping satisfying (8). Suppose that, for arbitrary $x,\ y \in K$, there exists $u \in K$ such that:*
(i) $||Ty - y||_2 \leq \beta||Tx - x||_2,\ 0 < \beta < 1;$
(ii) $||u - y||_1 \leq \mu||Tx - x||_2,\ \mu > 0.$
For $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the Ishikawa iteration defined by (4) with $\alpha_n \in [0, 1]$. Then, the Ishikawa iteration $\{x_n\}_{n=0}^{\infty}$ converges strongly to the fixed point of T.

Proof.

By going through similar process leading to (9) and using inequality condition (10) in (9), we obtain that

$$||x_{n+1} - p||_1 \leq [1 - (1-\delta)^2\alpha_n]||x_n - p||_1. \quad (15)$$

Using (14) in (15) yields

$$||x_{n+1} - p||_1 \leq [1 - (1-\delta)^2\alpha_n]\mu\beta^{n-1}||Tx_0 - x_0||_2 \to \ 0 \text{ as n} \to \infty,$$

from which we obtain again that $||x_{n+1} - p|| \to \ 0$ as n $\to \infty$,
that is, $\{x_n\}_{n=0}^{\infty}$ converges strongly to p.

In the sequel, we shall require the following definitions:

Definition 2.1 [1, 6]**:** Let $(X, ||.||)$ be a Banach space and K a nonempty closed convex subset of X. A mapping $T\colon E \to E$ is said to be $a-$*Lipschitzian* if there exists an $a \in [0, \infty)$ such that

$$||Tx - Ty|| \leq a||x - y||,\ \forall\ x,\ y \in K. \quad (17)$$

Definition 2.2: Let $(X, ||.||)$ be a Banach space and K a nonempty closed convex subset of X. A mapping $T\colon K \to K$ is said to be $(a, L)-$*Lipschitzian* if there exist an $a \in [0, \infty)$ and $L \geq 0$ such that

$$||Tx - Ty|| \leq L||x - Tx|| + a||x - y||,\ \forall\ x,\ y \in K. \quad (18)$$

Theorem 2.5. *Let E be a set on which two norms $||.||_1$ and $||.||_2$ are defined such that $(E,\ ||.||_1)$ is a Banach space, K a closed convex subset of E and $T\colon (K,\ ||.||_1) \to (K,\ ||.||_1)$ is an $(a, L)-$Lipschitzian mapping. Suppose that, for arbitrary $x,\ y \in K$, there exists $u \in K$ such that:*
(i) $||Ty - y||_2 \leq \beta||Tx - x||_2,\ 0 < \beta < 1;$
(ii) $||u - y||_1 \leq \mu||Tx - x||_2,\ \mu > 0.$
For $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the Mann iteration defined by (3) with $\alpha_n \in [0, 1]$. Then, the Mann iteration $\{x_n\}_{n=0}^{\infty}$ converges strongly to the fixed point of T.

Proof.
By (18), we have that

$$\begin{aligned}||x_{n+1}-p||_1 &\leq ||(1-\alpha_n)x_n+\alpha_n Tx_n-p||_1\\ &\leq (1-\alpha_n)||x_n-p||_1+\alpha_n||Tp-Tx_n||_1\\ &\leq (1-\alpha_n)||x_n-p||_1+a\alpha_n||p-x_n||_1\\ &= [1+(a-1)\alpha_n]||x_n-p||_1. \ (19)\end{aligned}$$

Again, using hypothesis (i) and (ii) leads to

$$||p-x_n||_1 \leq \mu\beta^{n-1}||Tx_0-x_0||_2. \tag{20}$$

Using (20) in (19) yields

$$||x_{n+1}-p||_1 \leq [1+(a-1)\alpha_n]\mu\beta^{n-1}||Tx_0-x_0||_2 \to \ 0 \text{ as n} \to \infty,$$

which thus gives $||x_{n+1}-p|| \to \ 0$ as n $\to \infty$, that is, $\{x_n\}_{n=0}^{\infty}$ converges strongly to p.

Theorem 2.6. *Let E be a set on which two norms $||.||_1$ and $||.||_2$ are defined such that $(E, \ ||.||_1)$ is a Banach space, K a closed convex subset of E and $T\colon (K, \ ||.||_1) \to (K, \ ||.||_1)$ is an $a-$Lipschitzian mapping. Suppose that, for arbitrary $x, \ y \in K$, there exists $u \in K$ such that:*
(i) $||Ty-y||_2 \leq \beta||Tx-x||_2, \ 0<\beta<1$;
(ii) $||u-y||_1 \leq \mu||Tx-x||_2, \ \mu>0$.
For $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the Mann iteration defined by (3) with $\alpha_n \in [0,1]$. Then, the Mann iteration $\{x_n\}_{n=0}^{\infty}$ converges strongly to the fixed point of T.

Proof: The proof is similar to that of Theorem 2.5 except that, in this case, $L=0$.

Remark 2.3: In Theorem 2.3 - Theorem 2.6, neither the condition $\sum_{n=0}^{\infty}\alpha_n=\infty$, nor, $0<\alpha\leq\alpha_n, \ \forall \ n$, is required for strong convergence of the Mann and Ishikawa iterations. Therefore, Theorem 2.3 - Theorem 2.6 are not just generalizations and extensions but in addition, they are improvements over those of Berinde [3, 4, 5] and some other previous results in the literature. Similar results as in Theorem 2.5 and Theorem 2.6 can be obtained for the Ishikawa iterative process too.

Remark 2.4: Pertaining to the contractivity conditions in (17) and (18), it is the usual practice to employ the restriction $a \in [0,1)$ for the type of convergence problem considered in this paper. However, it is now obvious from Theorem 2.5 and Theorem 2.6 that the restriction $a \in [0,1)$ can be extended to $a \in [0,\infty)$ without losing the assurance for the strong convergence of Mann and Ishikawa iterative processes. Thus, this is again, an improvement over the previous results in the literature.

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