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HIGHLY DISPERSIVE SOLITONS FOR STOCHASTIC PERTURBED COMPLEX GINZBURG-LANDAU EQUATION WITH EIGHTH-ORDER DISPERSION HAVING CUBIC-QUINTIC-SEPTIC-NONIC NONLINEARITY AND MULTIPLICATIVE WHITE NOISE

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Abstract. In this article, we investigate the complex Ginzburg-Landau (CGL) equation having eighth-order dispersion under the influence of multiplicative white noise, incorporating a cubic-quintic-septic-nonic nonlinear structure. The focus is on deriving highly dispersive soliton solutions using advanced analytical techniques, including the (F'/F)-expansion and improved Kudryashov approaches. The use of these techniques allows the development of specific soliton solutions, such as dark, bright, and singular solitons for specific sets of parametric values. Moreover, a study of the effect of stochastic noise on soliton dynamics has been conducted, offering an understanding of optical system dynamics in high-dispersion and noisy conditions. The results improve understanding of nonlinear wave behavior in optical fibers and other physical situations described by high-order CGL models.

1. Introduction

The Complex Ginzburg-Landau (CGL) equation is an essential mathematical representation of a broad class of nonlinear dynamical processes, and in particular nonlinear optics and wave phenomena. Its versatility enables the simulation of diverse solitonic behaviors in the presence of different dispersive and nonlinear influences. Some recent developments to this equation that include high-order dispersion and complex nonlinear behavior have also proved critical to optical systems operating under real and extreme conditions.

Several researchers have advanced the study of the CGL equation by exploring its solitonic solutions, nonlinear properties, and associated stochastic effects. For example, Zayed et al. [1] investigated the impact of eighth-order dispersion and cubic-quintic-septic-nonic nonlinearities in presence of stochastic noise to gain understanding of soliton dynamics. Kudryashov [2] and Zafar et al. [3] investigated

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the analytical solutions and the methods of analysis of the CGL equation, with the particular interest in nonlinear optics. Zhu et al. [4] have contributed to this area by the studying of bifurcations and the study of chaotic behavior, whereas Wang and Si [5] have considered solitonic structures due to the study of Kerr law nonlinearity.

Another major topic within the CGL framework has been the study of perturbative solutions (both rigorous and approximate) and conservation laws. Kudryashov [6] and Arnous et al. [7] investigated perturbative solutions, revealing diverse soliton dynamics. Zayed et al. [8, 9] generalizing this work to high-dispersion cases, focused on the interaction between nonlinear refractive indices and stochastic disturbances in optical fibers. Raza et al. [10] and Zayed et al. [11] extended this analysis to also cover quasi-periodic, bifurcation and chaotic structures in nonlinear systems, and therefore extended the range of soliton behaviour.

Advanced solution techniques have further enriched this field. The (F'/F)-expansion and Kudryashov techniques, as described by Zayed et al. [12, 13, 14, 16] They have also been essential for obtaining exact solutions to the CGL equation for many nonlinear and dispersive cases. Additionally, studies by Mohanty et al. [15] and Zayed et al. [17] extending these methods to other nonlinear settings, it has been discussed, and its general suitability and effectiveness have been emphasized.

In this paper, we study the eighth-order perturbed CGL equation involving cubic-quintic-septic-nonic nonlinearities and multiplicative white noise. Using the (F'/F)-expansion and more efficient Kudryashov methods, we obtain the explicit solitonic solutions, i.e., bright, dark and singular solitons, and study their dynamics in the presence of stochastic noise. This paper makes a contribution to the current evolution of theoretical models that are important to the design of complex optical systems.

In this work, we examine the CGL equation with eighth order high dispersion and multiplicative white noise effect in the Itô sense, which is coupled with cubic–quintic–septic–nonic nonlinear form [1]:

$$\begin{aligned} &i\phi_t + ia_1\phi_x + a_2\phi_{xx} + ia_3\phi_{xxx} + a_4\phi_{xxxx} + ia_5\phi_{xxxxx} + a_6\phi_{xxxxxx} \\ &+ia_7\phi_{xxxxxxx} + a_8\phi_{xxxxxxxx} + \left(b_1|\phi|^2 + b_2|\phi|^4 + b_3|\phi|^6 + b_4|\phi|^8\right)\phi \\ &+b_5\phi_{xt} + \sigma\left(\phi - ib_5\phi_x\right)\frac{dW(t)}{dt} = \frac{\beta}{4|\phi|^2\phi^*}\left[2|\phi|^2\left(|\phi|^2\right)_{xx} - \left\{\left(|\phi|^2\right)_x\right\}^2\right] \\ &+\alpha\frac{|\phi_x|^2}{\phi^*} + \gamma\phi + i\left[\lambda\left(|\phi|^2\phi\right)_x + \mu\left(|\phi|^2\right)_x\phi + \theta|\phi|^2\phi_x\right], \end{aligned} \tag{1.1}$$

let, $\phi(x,t)$ be a function that represents the wave profile which is a complex in nature. Also we let $\phi^*(x,t)$ stand for complex-conjugate of $\phi(x,t)$. The imaginary unit defined by i, where $i = \sqrt{-1}$. As for the initial component it is related to linear time variation. The coefficients a_j (where j goes from 1 to 8) are the constants related to different types of dispersion which including inter-modal dispersion (IMD), chromatic dispersion (CD) and higher-order dispersion (3OD, 4OD, 5OD, 6OD, 7OD, and 8OD). The coefficients b_j $(j = 1-4)$ are the constants

related to nonlinearity terms in the cubic, quintic, septic, and nonic laws. The coefficient b_5 is that which relates to the spatio-temporal (STD) variable. The coefficient σ is that which we use for the noise strength and $W(t)$ is the standard Wiener process. The derivative $\frac{dW(t)}{dt}$ is that we use for noise. The coefficients α and β are related to the nonlinear terms and λ is the detuning effect parameter. In short pulses the coefficient λ is which we use for self-steepening term also the coefficients μ and θ related to higher-order nonlinear dispersion.

In this model, the multiplicative white-noise term $\sigma\left(\phi - ib_5\phi_x\right)\frac{dW(t)}{dt}$ represents rapid, random fluctuations coming from the physical medium or external forcing. In many nonlinear optical systems, such noise does not act independently of the field, it depends on the instantaneous amplitude of the wave. Because of this dependence, the noise naturally enters the equation as a multiplicative term rather than as a simple additive disturbance. Physically, this term captures several realistic sources of randomness: variations in optical gain or loss, small changes in the refractive index, pump-power instability in amplifiers, and thermal or acoustic vibrations within the fiber core. In semiconductor waveguides, it also reflects fluctuations in carrier density that directly modulate the wave amplitude. These microscopic, fast processes create stochastic changes in both amplitude and phase, and the multiplicative structure allows the model to reflect how stronger fields experience proportionally stronger noise. Consequently, this noise term provides a realistic description of how environmental and material randomness influences soliton behavior, particularly in regimes where dispersion and nonlinearity are highly sensitive to small perturbations.

2. Converting Eq. (1.1) to ordinary differential equations

The following solution structure must be implemented in order to solve Eq. (1.1).

$$\phi\left(x,t\right) = q\left(\eta\right)e^{i\left(-kx+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}, \tag{2.1}$$

Here, the amplitude component of the soliton solution is described by the real-valued function $q\left(\eta\right)$. The wave number is represented by ω, whereas the frequency of the solitons is represented by the parameter k. Additionally, θ_0 indicates the phase constant, and σ indicates the severity of the noise. The wave variable η is provided by

$$\eta = k\left(x - \upsilon t\right), \tag{2.2}$$

where the wave width and the soliton's speed are denoted by the nonzero constants k and υ respectively. The real portion may be obtained by decomposing Eq. (2.1) into Eq. (1.1) and breaking it down into real and imaginary components, the real part is as follows:

$$\begin{gathered}
a_8k^8q^{(8)}q + k^6\left(28a_8k^2 + a_6\right)q^{(6)}q + k^4\left(5k\left[a_5 - 3k\left\{14a_8k^2 + a_6\right\}\right] + a_4\right)q^{(4)}q \\
-k^2\left(-140a_8k^6 - 15a_6k^4 + 10a_5k^3 + 6a_4k^2 - 3a_3k - a_2 + b_5\upsilon + \beta\right)qq'' - \alpha k^2{q'}^2 \\
+\left(-7a_8k^8 - a_6k^6 + a_5k^5 + a_4k^4 - a_3k^3 - a_2k^2 + a_1k + b_5k\sigma^2\right. \\
\left.+b_5k\omega - \gamma + \sigma^2 - \omega\right)q^2 + q^4\left(b_1 + 2k\left[\lambda + \mu\right]\right) + b_4q^{10} + b_3q^8 + b_2q^6 = 0,
\end{gathered} \tag{2.3}$$

and the imaginary part as:

$$kqq'\left(8a_8k^7-7a_7k^6-6a_6k^5+4a_4k^3-3a_3k^2-2a_2k+a_1-b_5\sigma^2+b_5k\upsilon+b_5\omega-\upsilon\right)$$
$$+k^7\left(a_7-8a_8k\right)qq^{(7)}+k^5\left\{7k^2\left(8a_8k-3a_7\right)-6ka_6+a_5\right\}qq^{(5)}-kq^3\left(\theta+3\lambda+2\mu\right)q'$$
$$k^3\left(k^2\left\{-56a_8k^3+35a_7k^2+20a_6k-10a_5\right\}-4ka_4+a_3\right)qq^{(3)}=0. \tag{2.4}$$

From the imaginary part, the soliton speed is reached

$$\upsilon=\frac{k^2\left(48a_8k^5+6a_6k^3-5a_5k^2-4a_4k+3a_3\right)+2a_2k-a_1+b_5\left(\sigma^2-\omega\right)}{b_5k-1}, \tag{2.5}$$

and the soliton frequency

$$k=\frac{a_7}{8a_8}, \tag{2.6}$$

with parametric restrictions

$$a_3=\frac{a_7\left(7a_7^4+20a_6a_8a_7^2+128a_4a_8^3\right)}{256a_8^4}, \tag{2.7}$$

$$a_5=\frac{a_7\left(7a_7^2+24a_6a_8\right)}{32a_8^2}, \tag{2.8}$$

$$\theta+3\lambda+2\mu=0. \tag{2.9}$$

Eq. (2.3) can be simplified to

$$\vartheta_9q^{(6)}q+\vartheta_8q^{(4)}q+\vartheta_7qq''+\vartheta_6q'^2+\vartheta_5q^{10}+\vartheta_4q^8+\vartheta_3q^6$$
$$+\vartheta_2q^4+\vartheta_1q^2+k^2q^{(8)}q=0, \tag{2.10}$$

with

$$\vartheta_1=-\frac{7a_8k^8+a_6k^6-a_5k^5+a_3k^3+ak^2-a_1k+b_5k\omega+\gamma-\sigma^2+\omega}{a_8k^6},\ \vartheta_2=-\frac{b_1+2k(\lambda+\mu)}{a_8k^6},\ \vartheta_3=-\frac{b_2}{a_8k^6},$$
$$\vartheta_4=\frac{b_3}{a_8k^6},\ \vartheta_5=\frac{b_4}{a_8k^6},\ \vartheta_6=-\frac{a}{a_8k^4},\ \vartheta_7=\frac{140a_8k^6+15a_6k^4-10a_5k^3-6a_4k^2+3a_3k+a_2-b_5\upsilon-\beta}{a_8k^4},$$
$$\vartheta_8=\frac{5k\left(a_5-3k\left(14a_8k^2+a_6\right)\right)+a_4}{a_8k^2},\ \vartheta_9=\frac{a_6}{a_8}+28k^2, \tag{2.11}$$

where $a_8\neq0, k\neq0, a_7\neq0$ and $b_5k\neq1$.
Next, we will apply the (F'/F)-expansion approach and an improved to Kudryashov's approach to construct solitons waves solutions of Eq. (1.1).

3. The (F'/F)-expansion approach

For solving Eq. (2.10), balancing the terms $q^{(8)}q$ and q^{10}, gives $N=1$. The (F'/F) expansion approach described in [12-15] suposes the solution of Eq. (2.10) takes the form:

$$q(\eta)=\zeta_0+\zeta_1\left[\frac{F'(\eta)}{F(\eta)}\right],\ \zeta_1\neq0. \tag{3.1}$$

here $F(\eta)$ is a function of η satisfying the ODE:

$$F''(\eta)+\lambda F'(\eta)+\Omega F(\eta)=0, \tag{3.2}$$

where $\zeta_0,\ \zeta_1,\ \lambda$ and Ω are constants. The general solution of Eq. (3.2), obtains three cases of $\frac{F'(\eta)}{F(\eta)}$ which reported in [12] as follows:

$$\frac{F'}{F}=\begin{cases} -\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\Omega}}{2}\left[\frac{K_1\sinh\left(\frac{\eta}{2}\sqrt{\lambda^2-4\Omega}\right)+K_2\cosh\left(\frac{\eta}{2}\sqrt{\lambda^2-4\Omega}\right)}{K_1\cosh\left(\frac{\eta}{2}\sqrt{\lambda^2-4\Omega}\right)+K_2\sinh\left(\frac{\eta}{2}\sqrt{\lambda^2-4\Omega}\right)}\right], & \lambda^2-4\Omega>0,\\ -\frac{\lambda}{2}+\frac{\sqrt{-(\lambda^2-4\Omega)}}{2}\left[\frac{K_1\cos\left(\frac{\eta}{2}\sqrt{-(\lambda^2-4\Omega)}\right)-K_2\sin\left(\frac{\eta}{2}\sqrt{-(\lambda^2-4\Omega)}\right)}{K_1\sin\left(\frac{\eta}{2}\sqrt{-(\lambda^2-4\Omega)}\right)+K_2\cos\left(\frac{\eta}{2}\sqrt{-(\lambda^2-4\Omega)}\right)}\right], & \lambda^2-4\Omega<0,\\ -\frac{\lambda}{2}+\frac{K_2}{K_1+K_2\eta}, & \lambda^2-4\Omega=0. \end{cases} \tag{3.3}$$

In this article, we focus only on soliton solutions. For this aim, let $\lambda^2-4\Omega>0$, and using Maple for evaluating (3.1) with along Eq. (3.2) into Eq. (2.10), ones can be arrive the following result:

$$\zeta_0=\frac{\lambda\sqrt[8]{-\frac{40\,320}{\vartheta_9}}}{2},\ \zeta_1=\sqrt[8]{-\frac{40\,320}{\vartheta_9}}, \tag{3.4}$$

via the restrictions

$$\begin{aligned}
\vartheta_1&=\frac{-8\left(\mu-\frac{\lambda^2}{4}\right)\vartheta_6\sqrt[4]{-\frac{40\,320}{\vartheta_9}}-1140\lambda^8+18240\lambda^6\mu-109440\lambda^4\mu^2+\left(291840\mu^3-6\vartheta_3\right)\lambda^2-291840\mu^4+24\mu\vartheta_3+20\vartheta_5}{69(-\lambda^2+4\mu)^3},\\
\vartheta_4&=0,\\
\vartheta_2&=\frac{1}{138(-\lambda^2+4\mu)^2}\left[68\left(\mu-\frac{\lambda^2}{4}\right)\vartheta_6\sqrt[4]{-\frac{40\,320}{\vartheta_9}}+5412\lambda^8-86592\lambda^6\mu\right.\\
&\left.+519552\lambda^4\mu^2+\left(-1385472\mu^3+120\vartheta_3\right)\lambda^2+1385472\mu^4-480\mu\vartheta_3-308\vartheta_5\right],\\
\vartheta_7&=\frac{1}{138\sqrt{-\frac{70}{\vartheta_9}}(\lambda^2-4\mu)^2}\left[-11880\lambda^8+190080\lambda^6\mu-1140480\lambda^4\mu^2+3041280\lambda^2\mu^3\right.\\
&\left.-3041280\mu^4+90\lambda^2\vartheta_3-360\mu\vartheta_3-392\vartheta_5-53\sqrt[4]{-\frac{40\,320}{\vartheta_9}}\vartheta_6\left(-4\mu+\lambda^2\right)\right],\\
\vartheta_8&=-\frac{\sqrt[4]{-\frac{40\,320}{\vartheta_9}}\vartheta_9}{1932(\lambda^2-4\mu)^3}\left[879\lambda^8+84384\lambda^4\mu^2-14064\lambda^6\mu-225024\lambda^2\mu^3\right.\\
&\left.+225024\mu^4-3\lambda^2\vartheta_3+12\mu\vartheta_3+10\vartheta_5+\sqrt[4]{-\frac{40\,320}{\vartheta_9}}\vartheta_6\left(-4\mu+\lambda^2\right)\right].
\end{aligned} \tag{3.5}$$

Substituting (3.4) along with (3.3) into (3.1), the general solution of Eq. (2.10)

$$\phi(x,t)=\frac{\sqrt{\lambda^2-4\mu}\sqrt[8]{-\frac{40\,320}{\vartheta_9}}}{2}\left\{\frac{K_1\sinh\left(\frac{\eta\sqrt{\lambda^2-4\mu}}{2}\right)+K_2\cosh\left(\frac{\eta\sqrt{\lambda^2-4\mu}}{2}\right)}{K_1\cosh\left(\frac{\eta\sqrt{\lambda^2-4\mu}}{2}\right)+K_2\sinh\left(\frac{\eta\sqrt{\lambda^2-4\mu}}{2}\right)}\right\}e^{i\left(-kx+\omega t+\sigma W(t)-\sigma^2t+\theta_0\right)}, \tag{3.6}$$

In the general solution (3.6), if $K_2=0$, $K_1\neq0$ and $\mu=0$, we get the dark soliton

$$\phi(x,t)=\frac{\lambda\sqrt[8]{-\frac{40\,320}{\vartheta_9}}}{2}\tanh\left(\frac{\lambda}{2}\eta\right)e^{i\left(-kx+\omega t+\sigma W(t)-\sigma^2t+\theta_0\right)}, \tag{3.7}$$

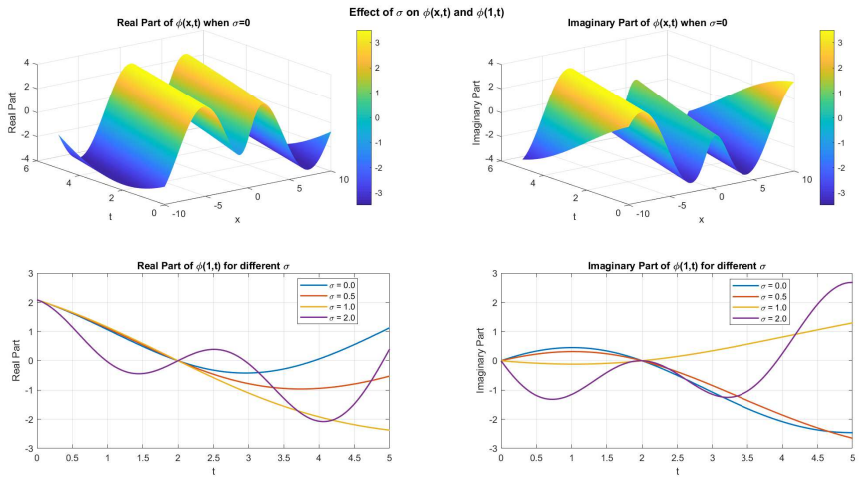

FIGURE 1. Dark soliton solution of Eq. (3.7) with the parameters $\lambda = 2.75$, $\mu = 0$, $K_1 = 1$, $K_2 = 0$, $k = 0.5$, $\omega = 0.4$, $\theta_0 = 0.5$, $a_6 = -15$, $a_8 = 0.5$, $x \in [-10, 10]$, and $t \in [0, 6]$.

While if $K_2 \neq 0$, $K_1 = 0$ and $\mu = 0$, we have the singular soliton

$$\phi(x,t) = \frac{\lambda \sqrt[8]{-\frac{40\,320}{\vartheta_9}}}{2} \coth\left(\frac{\lambda}{2}\eta\right) e^{i\left(-kx+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}, \tag{3.8}$$

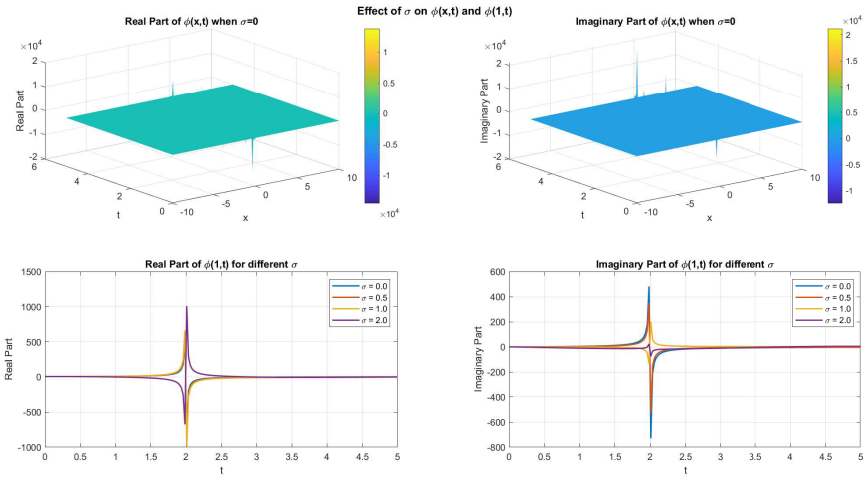

FIGURE 2. Singular soliton solution of Eq. (3.8) with the parameters $\lambda = 2.75$, $\mu = 0$, $K_1 = 0$, $K_2 = 0.01$, $k = 0.5$, $\omega = 0.4$, $\theta_0 = 0.5$, $a_6 = -15$, $a_8 = 0.5$, $x \in [-10, 10]$, and $t \in [0, 6]$.

The solutions (3.6)-(3.8) exist under the conditions given by (3.5).

4. Outline of the Improved Kudryashov Method

The improved Kudryashov method is a refined analytical technique designed to derive exact travelingwave solutions of high-order nonlinear differential equations. Compared with the classical Kudryashov approach, this enhanced version introduces a more flexible auxiliary function and a generalized polynomial ansatz. These improvements allow the method to treat strong nonlinearities, high-order dispersion, and stochastic modulation more effectively. A concise outline of the procedure is presented below.

(1) **Traveling-wave transformation.** The governing PDE is reduced to an ODE by applying the transformation

$$\phi(x,t) = q(\eta)\, e^{i(-kx+\omega t+\sigma W(t)-\sigma^2 t+\theta_0)}, \qquad \eta = k(x-vt),$$

which yields a nonlinear ODE of the form

$$\mathcal{P}(q, q', q'', \ldots, q^{(8)}) = 0.$$

This step extracts the wave profile $q(\eta)$ and converts the PDE dynamics into a tractable ODE framework.

(2) **Auxiliary function.** An auxiliary function $H(\eta)$ is introduced and is assumed to satisfy

$$H'(\eta)^2 = H(\eta)^2\Big(1-\kappa H^{2r}(\eta)\Big)\,\ln^2 K,$$

where $\kappa \neq 0$, $K \neq 1$, and r is a positive integer. Depending on the parameter choices, this nonlinear relation produces hyperbolic, rational, or mixed-function forms.

(3) **Generalized polynomial ansatz.** The soliton profile $q(\eta)$ is expanded as a finite polynomial in $H(\eta)$:

$$q(\eta) = \sum_{i=0}^{N} L_i\, H^i(\eta), \qquad L_N \neq 0.$$

The polynomial degree N is determined by balancing the dominant nonlinear term (e.g., q^{10}) with the highest-order dispersive term (e.g., $q^{(8)}$). For the present model, this yields two primary cases:

$$r = 1 \Rightarrow N = 1 \quad \text{(Type I)}, \qquad r = 2 \Rightarrow N = 2 \quad \text{(Type II)}.$$

(4) **Substitution and coefficient matching.** Substituting the ansatz and auxiliary relation into the reduced equation $\mathcal{P} = 0$, collecting like powers of $H(\eta)$, and setting each coefficient to zero produces an algebraic system involving L_i, κ, K, and the reduced coefficients ϑ_i. Solving this system yields the parameter constraints associated with the exact solutions, such as those presented in Eqs. (4.6) and (4.13).

(5) **Construction of explicit solitary waves.** Once the auxiliary function is determined, explicit traveling-wave solutions follow. A representative form of $H(\eta)$ is

$$H(\eta) = \frac{4S\ln(K)}{4S^2K^{\eta} + \kappa K^{-\eta}},$$

from which the polynomial ansatz produces closed-form soliton solutions. Choosing $\kappa = 4S^2$ yields the standard bright sech-type solitons, while other parameter selections generate dark-type, singular, or multi-humped wave structures.

5. Utilizing an improved Kudryashov-approach

For solving Eq. (2.10) utilizing an improved Kudryashov-approach [12, 16, 17], The solution takes the form:

$$q(\eta) = \sum_{i=0}^{N} L_i H^i(\eta), \tag{4.1}$$

where L_i $(i = 0, 1, 2, ..., N)$ are constants such that $L_N \neq 0$. The function $H(\eta)$ satisfies the nonlinear ODE:

$$H'^2(\eta) = H^2(\eta)\left[1 - \varkappa H^{2r}(\eta)\right]\ln^2 K,\ 0 < K \neq 1, \tag{4.2}$$

where $\varkappa$ is a non-zero constant and r is an integer number. Balancing $q(\eta)q^{(8)}(\eta)$ with $q^{10}(\eta)$ in Eq. (2.10) gives $2N + 8r = 10N \Longrightarrow N = r$.
Next, we consider the two typs of r as follows:
Type-I. If $r = 1$, the formula solution (4.1) becomes:

$$q(\eta) = L_0 + L_1 H(\eta),\ L_1 \neq 0, \tag{4.3}$$

with the auxiliary equation:

$$H'^2(\eta) = H^2(\eta)\left[1 - \varkappa H^2(\eta)\right]\ln^2 K. \tag{4.4}$$

Evaluating (4.3) with (4.4) into Eq. (2.10) and using Maple, the following result can be found:

$$L_0 = 0,\ L_1 = \left(-\tfrac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{8}}\ln(K), \tag{4.5}$$

via the restrictions

$$\begin{cases} \vartheta_4 = \frac{-1640\ln(K)^6\varkappa - 182\ln(K)^4\vartheta_1\varkappa - 20\ln(K)^2\vartheta_2\varkappa + \left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{4}}\vartheta_6 - 2\vartheta_3\varkappa}{\varkappa}, \\ \vartheta_5 = \frac{-\ln(K)^2\vartheta_6\left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{4}} + 1639\ln(K)^2\varkappa\left(\ln(K)^6 + \frac{181\ln(K)^4\vartheta_1}{1639} + \frac{19\ln(K)^2\vartheta_2}{1639} + \frac{\vartheta_3}{1639}\right)}{\varkappa}, \\ \vartheta_7 = \frac{\sqrt{-\frac{70\varkappa^4}{\vartheta_9}}\left(966\ln(K)^4 + 35\ln(K)^2\vartheta_1 + \vartheta_2\right)\vartheta_9}{70\varkappa^2}, \\ \vartheta_8 = -\frac{\left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{4}}\left(84\ln(K)^2 + \vartheta_1\right)\vartheta_9}{56\varkappa}, \end{cases} \tag{4.6}$$

provided $\vartheta_9 < 0$.
By substituting the result (4.5) with the exact solutions of Eq. (4.4) as reported in [16], the exact solution for Eq. (1.1) takes the form:

$$\phi(x,t) = \left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{8}}\left(\frac{4S\ln(K)}{4S^2K^{\eta} + \varkappa K^{-\eta}}\right)e^{i\left(-kx + \omega t + \sigma W(t) - \sigma^2 t + \theta_0\right)}, \tag{4.7}$$

where S is a nonzero constant. The exact solution (4.7) can be expressed as the solitary wave solution:

$$\phi(x,t) = \left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{8}}\left(\frac{4S\ln(K)}{(4S^2+\varkappa)\cosh(\eta\ln K) + (4S^2-\varkappa)\sinh(\eta\ln K)}\right)e^{i\left(-kx + \omega t + \sigma W(t) - \sigma^2 t + \theta_0\right)}, \tag{4.8}$$

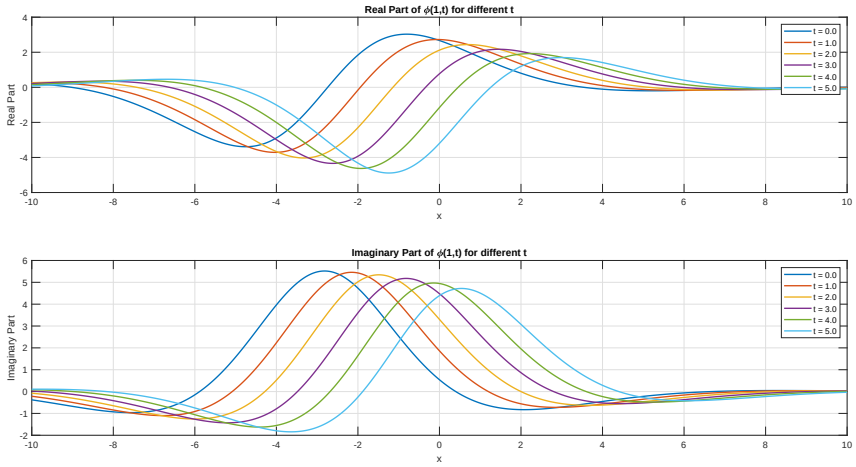

FIGURE 3. Solitary-wave soliton solution of Eq. (4.8) with the parameters $K = 2.5$, $k = 0.5$, $\omega = 0.4$, $\theta_0 = 0.2$, $a_6 = -15$, $a_8 = 0.5$, $\varkappa = 0.1$, $S = 0.6$, $\sigma = 0$ and $x \in [-10, 10]$.

In particular, If $\varkappa = 4S^2$, the solitary-wave solution (4.8) can be formulated as the bright soliton

$$\phi(x,t) = \left(-\frac{40320}{\vartheta_9}\right)^{\frac{1}{8}} \ln(K)\,\mathrm{sech}(\eta \ln K)\, e^{i\left(-kx+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \tag{4.9}$$

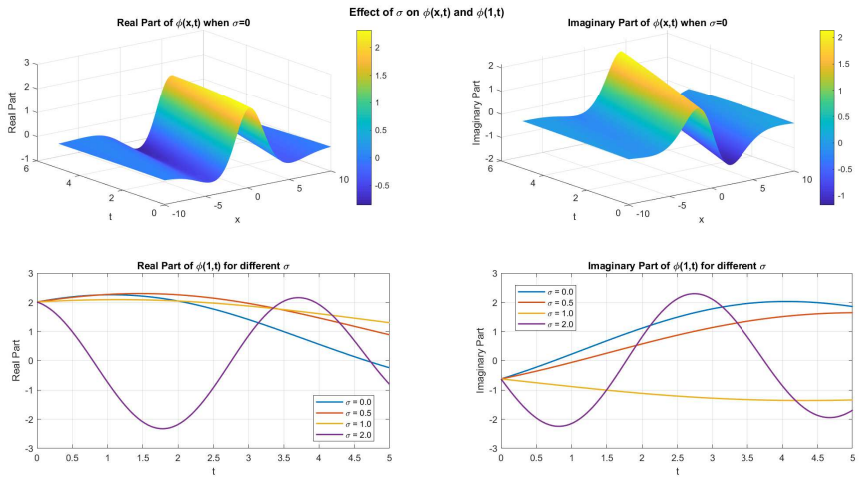

FIGURE 4. Bright soliton solution of Eq. (4.9) with the parameters $K = 2.5$, $k = 0.5$, $\omega = 0.4$, $\theta_0 = 0.2$, $a_6 = -15$, $a_8 = 0.5$, $x \in [-10, 10]$ and $t \in [0, 6]$.

The solutions (4.7)-(4.9) exist under the conditions given by (4.6).

Type-II. If $r = 2$, the formal solution of Eq (4.1) becomes:

$$q(\eta) = L_0 + L_1 H(\eta) + L_2 H^2(\eta), \ L_2 \neq 0, \tag{4.10}$$

via the auxiliary equation:

$$H'^2(\eta) = H^2(\eta)\left[1 - \varkappa H^4(\eta)\right] \ln^2 K. \tag{4.11}$$

By evaluating (4.10) along with (4.11) into Eq. (2.10) and using Maple, the following result can be obtained:

$$L_0 = 0, \ L_1 = 0, \ L_2 = 2\left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{8}} \ln(K), \tag{4.12}$$

via the restrictions

$$\begin{cases} \vartheta_3 = -\dfrac{104960\ln(K)^6\varkappa + 2912\ln(K)^4\vartheta_1\varkappa + 80\ln(K)^2\vartheta_2\varkappa - \left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{4}}\vartheta_6 + \vartheta_4\varkappa}{2\varkappa}, \\ \vartheta_5 = \dfrac{2\ln(K)^2\left(104832\ln(K)^6\varkappa + 2880\ln(K)^4\vartheta_1\varkappa + 72\ln(K)^2\vartheta_2\varkappa - \left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{4}}\vartheta_6 - \vartheta_4\varkappa\right)}{\varkappa}, \\ \vartheta_7 = \dfrac{\sqrt{-\frac{70\varkappa^4}{\vartheta_9}}\left(15456\ln(K)^4 + 140\ln(K)^2\vartheta_1 + \vartheta_2\right)\vartheta_9}{70\varkappa^2}, \\ \vartheta_8 = -\dfrac{\left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{4}}\left(336\ln(K)^2 + \vartheta_1\right)\vartheta_9}{56\varkappa}. \end{cases} \tag{4.13}$$

provided $\vartheta_9 < 0$.
By incorporating the well-known solutions of Eq. (4.11) obtained in [16] and substituting (4.12) into (4.10), the exact solution can be obtained as follows:

$$\phi(x,t) = \left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{8}} \left(\frac{8S\ln(K)}{4S^2K^{2\eta}+\varkappa K^{-2\eta}}\right) e^{i\left(-kx+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}, \tag{4.14}$$

The exact solution (4.14) can be reconstructed as the solitary solution

$$\phi(x,t) = \left(-\frac{40320\varkappa^4}{\vartheta_9}\right)^{\frac{1}{8}} \left(\frac{8S\ln(K)}{(4S^2+\varkappa)\cosh(2\eta\ln K)+(4S^2-\varkappa)\sinh(2\eta\ln K)}\right) e^{i\left(-kx+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}, \tag{4.15}$$

In particular, If $\varkappa = 4S^2$, the solitary solution (4.15) can be formulated as the bright soliton

$$\phi(x,t) = \left(-\frac{10321920}{\vartheta_9}\right)^{\frac{1}{8}} \ln(K)\,\mathrm{sech}\,(2\eta\ln K)\, e^{i\left(-kx+\omega t+\sigma W(t)-\sigma^2 t+\theta_0\right)}. \tag{4.16}$$

The solutions (4.14)-(4.16) exist under the conditions given by (4.13).
Finally, by adopting a similar approach, a multitude of additional solutions for equation (1.1) can be discovered by varying the parameter r. For the sake of simplicity, these values are not included here.

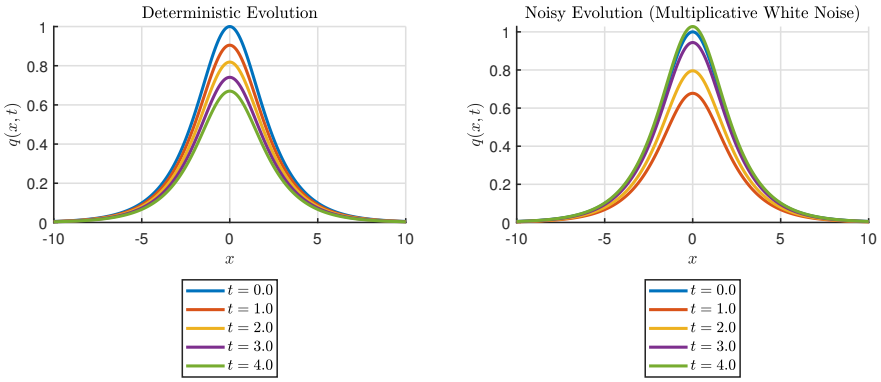

FIGURE 5. Deterministic versus noisy evolution of a bright soliton. The left panel shows the purely deterministic evolution governed by $\mathrm{d}q = -\alpha q\,\mathrm{d}t$, where the soliton remains smooth, symmetric, and well-localized. In contrast, the right panel depicts the effect of multiplicative white noise, $\mathrm{d}q = -\alpha q\,\mathrm{d}t + \sigma q\,\mathrm{d}W_t$, which induces amplitude jitter, asymmetry, spectral broadening, and progressive distortion. This comparison highlights how stochastic forcing destabilizes bright solitons in the eighth-order CGL framework.

6. Results and Discussion

Figures 1–4 present the soliton solutions obtained for the stochastic perturbed complex Ginzburg–Landau equation with eighth-order dispersion under different

parametric conditions. Figure 1 illustrates the dark soliton structure, characterized by a localized intensity dip that remains stable under the imposed high-order dispersion parameters ($\lambda = 2.75$, $\mu = 0$). The hyperbolic tangent profile confirms its robustness and symmetry, even in the presence of stochastic perturbations. Figure 2 displays the singular soliton solution, whose hyperbolic cotangent behavior exhibits a pronounced central singularity, demonstrating high sensitivity to dispersion and nonlinear effects. Figure 3 shows a solitary-wave solution constructed using the improved Kudryashov method; the real and imaginary components evolve smoothly, revealing the intricate interplay between eighth-order dispersion and weak stochastic modulation ($\kappa = 0.1$, $S = 0.6$). In Figure 4, a bright soliton with a symmetric and well-localized amplitude peak is obtained, indicating stability under strong dispersive effects when $\kappa = 4S^2$.

A more detailed comparison between deterministic and stochastic soliton dynamics is provided in Figure 5, which constitutes an important addition to the analysis. The left panel presents the deterministic evolution governed by the decay law $dq = -\alpha q\, dt$, where the soliton preserves its smoothness, symmetry, and spatial localization. In this regime, amplitude decay is uniform and the waveform exhibits no structural distortion, reflecting the predictable behavior typical of noiseless eighth-order CGL systems.

In contrast, the right panel of Figure 5 shows the evolution under multiplicative white noise, $dq = -\alpha q\, dt + \sigma q\, dW_t$. The stochastic forcing induces amplitude jitter, phase fluctuations, asymmetry in the envelope, and spectral broadening. These distortions accumulate over time, demonstrating that multiplicative noise couples directly to the amplitude of the wave, thereby destabilizing the soliton. This behavior highlights the nonlinear interaction between noise and high-order dispersion, confirming that soliton coherence and localization degrade in stochastic environments. Such effects have significant implications for optical-fiber signal propagation, noise-managed communication systems, and the design of soliton-based devices in turbulent media.

Collectively, the results shown in Figures 1–5 reveal the rich dynamical features of eighth-order CGL solitons and emphasize the critical role of stochastic forcing in shaping their stability, symmetry, and long-term evolution.

TABLE 1. Summary of existence and stability conditions for the soliton solutions.

Category	**Condition**	**Meaning / Stability Implication**
Dispersion constraints	$a_8 \neq 0,\ a_7 \neq 0$	Ensures non-degenerate eighth-order dispersion.
	$k = \dfrac{a_7}{8a_8}$	Fixes the soliton wave number and avoids phase mismatch.
	$b_5 k \neq 1$	Prevents singularities in the soliton velocity expression.
Compatibility conditions	a_3, a_5 as in Eqs. (2.7)–(2.8)	Ensures consistency of the reduced ODE and removes incompatible imaginary contributions.
	$\theta + 3\lambda + 2\mu = 0$	Balances higher-order nonlinear terms and self-steepening effects.
Hyperbolic-soliton existence	$\lambda^2 - 4\mu > 0$	Guarantees real tanh, coth, and sech profiles; ensures localization.
	$k \neq 0$	Prevents degeneracy of the traveling-wave frame.
Dark soliton branch	$K_2 = 0,\ K_1 \neq 0,\ \mu = 0$	Produces a stable tanh-shaped dark soliton.
Singular soliton branch	$K_1 = 0,\ K_2 \neq 0,\ \mu = 0$	Generates a coth profile with a central singularity.
Bright soliton branch	$\kappa = 4S^2$	Ensures symmetric sech-type bright soliton formation.
Kudryashov existence condition	$\vartheta_9 < 0$	Required for real solitary-wave solutions in the auxiliary ODE.
General regularity	Parameters from (3.4), (4.5), (4.12) real	Ensures physically meaningful (non-complex) amplitudes.

6. **Conclusions**

In this work, we constructed and analyzed explicit bright, dark, and singular soliton solutions of the eighth-order complex GinzburgLandau equation incorporating cubicquinticsepticnonic nonlinearities and multiplicative white noise. By employing the (F'/F)-expansion technique and the improved Kudryashov method, we obtained a diverse set of localized wave structures that reveal how high-order dispersion and nonlinear refractive effects shape soliton behavior in complex optical settings. A central outcome of this study is the clear contrast between deterministic and stochastic soliton evolution, highlighted in Figure 5. Under deterministic conditions, solitons retain their smoothness, symmetry, and spatial localization. However, when multiplicative noise is introduced, the dynamics change significantly: amplitude fluctuations, envelope asymmetry, and gradual waveform distortion all emerge. These observations show that stochastic forcing interacts in a strongly nonlinear manner with high-order dispersion, leading to notable changes in soliton stability and long-term coherence. The analytical results and physical insights presented here provide a useful foundation for designing optical systems that must function reliably in noisy or turbulence-dominated environments. Moreover, the

derived parameter constraints offer a structured approach for predicting the existence and stability of solitons in broader classes of nonlinear stochastic wave models. Overall, the findings of this work contribute to future theoretical advances and practical developments in nonlinear optics, high-order dispersive media, and stochastic wave dynamics.

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