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ROUGH CONVERGENCE IN 2-NORMED SPACES

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Abstract. In this work, we introduced the notions of rough convergence, rough Cauchy sequence and the set of rough limit points of a sequence and obtained rough convergence criteria associated with this set in 2-normed space. Later, we proved that this set is closed and convex. Finally, we examined the relations between rough convergence and rough Cauchy sequence in 2-normed space.

1. Introduction

The concept of 2-normed spaces was initially introduced by Gähler [8, 9] in the 1960's. Since then, this concept has been studied by many authors. Gürdal and Pehlivan [13] studied statistical convergence, statistical Cauchy sequence and investigated some properties of statistical convergence in 2-normed spaces. Şahiner et al. [24] and Gürdal [15] studied $\mathcal{I}$-convergence in 2-normed spaces. Gürdal and Açık [14] investigated $\mathcal{I}$-Cauchy and $\mathcal{I}^*$-Cauchy sequences in 2-normed spaces. Sarabadan and Talebi [20] studied statistical convergence and ideal convergence of sequences of functions in 2-normed spaces. Arslan and Dündar [1] investigated the concepts of $\mathcal{I}$-convergence, $\mathcal{I}^*$-convergence, $\mathcal{I}$-Cauchy and $\mathcal{I}^*$-Cauchy sequences of functions in 2-normed spaces. Also, Yegül and Dündar [25] studied statistical convergence of sequence of functions in 2-normed spaces. Futhermore, a lot of development have been made in this area (see [5, 16, 21, 23]).

The idea of rough convergence was first introduced by Phu [17] in finite - dimensional normed spaces. In [17], he showed that the set $\mathtt{LIM}^r x_i$ is bounded, closed, and convex; and he introduced the notion of rough Cauchy sequence. He also investigated the relations between rough convergence and other convergence types and the dependence of $\mathtt{LIM}^r x_i$ on the roughness degree r. In another paper [18] related to this subject, he defined the rough continuity of linear operators and showed that every linear operator $f : X \to Y$ is r -continuous at every point $x \in X$ under the assumption $dimY < \infty$ and $r > 0$ where X and Y are normed spaces. In [19], he extended the results given in [17] to infinite-dimensional normed spaces. Aytar [3] studied of rough statistical convergence and defined the set of rough statistical limit points of a sequence and obtained two statistical convergence criteria associated with this set and prove that this set is closed and convex. Also, Aytar [4]

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studied that the r-limit set of the sequence is equal to the intersection of these sets and that r-core of the sequence is equal to the union of these sets. Recently, Dündar and Çakan [6, 7] introduced the notion of rough $\mathcal{I}$-convergence and the set of rough $\mathcal{I}$-limit points of a sequence and studied the notion of rough convergence and the set of rough limit points of a double sequence.

In this paper, using the concept of rough convergence and concept of 2-normed spaces, we introduce the notion of rough convergence in 2-normed spaces. Defining the set of rough limit points of a sequence, we obtain two convergence criteria associated with this set. Later, we prove that this set is closed and convex. Finally, we examine the relations between the set of cluster points and the set of rough limit points of a sequence. We note that our results and proof techniques presented in this paper are analogues of those in Phu's [17] paper. Namely, the actual origin of most of these results and proof techniques is them papers. The following our theorems and results are the extension of theorems and results in [17].

Now, we recall the concept of 2-normed space, ideal convergence and some fundamental definitions and notations (See [10, 11, 12, 13, 14, 15, 20, 24]).

Let X be a real vector space of dimension d, where $2 \leq d < \infty$. A 2-norm on X is a function $\|\cdot,\cdot\| : X \times X \to \mathbb{R}$ which satisfies the following statements:

(i) $\|x, y\| = 0$ if and only if x and y are linearly dependent.
(ii) $\|x, y\| = \|y, x\|$.
(iii) $\|\alpha x, y\| = |\alpha| \|x, y\|$, $\alpha \in \mathbb{R}$.
(iv) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$.

As an example of a 2-normed space we may take $X = \mathbb{R}^2$ being equipped with the 2-norm
$\|x, y\| :=$ the area of the parallelogram based on the vectors x and y
which may be given explicitly by the formula

$$\|x, y\| = |x_1y_2 - x_2y_1|; \quad x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2.$$

In this study, we suppose X to be a 2-normed space having dimension d; where $2 \leq d < \infty$. The pair $(X, \|\cdot,\cdot\|)$ is then called a 2-normed space.

A sequence (x_n) in 2-normed space $(X, \|\cdot,\cdot\|)$ is said to be convergent to L in X if

$$\lim_{n\to\infty} \|x_n - L, y\| = 0,$$

for every $y \in X$. In such a case, we write $\lim_{n\to\infty} x_n = L$ and call L the limit of (x_n).

Example 1.1. *Let $x = (x_n) = (\frac{n}{n+1}, \frac{1}{n})$, $L = (1, 0)$ and $z = (z_1, z_2)$. It is clear that (x_n) convergent to $L = (1, 0)$ in 2-normed space.*

Let (x_n) be a sequence in $(X, \|.,.\|)$ 2-normed space. If for $\forall \varepsilon > 0$, there is an $N = N(\varepsilon) \in \mathbb{N}$ such that for $\forall m, n \geq N$ and for every $z \in X$,

$$\|x_m - x_n, z\| < \varepsilon$$

then, (x_n) said to be a Cauchy sequence in $(X, \|.,.\|)$.

Let r be a nonnegative real number and $\mathbb{R}^n$ denotes the real n-dimensional space with the norm $\|.\|$. Consider a sequence $x = (x_i) \subset \mathbb{R}^n$.

The sequence $x = (x_i)$ is said to be r-convergent to x_*, denoted by $x_i \xrightarrow{r} x_*$ provided that

$$\forall \varepsilon > 0 \ \exists i_\varepsilon \in \mathbb{N} : \ i \geq i_\varepsilon \Rightarrow \|x_i - x_*\| < r + \varepsilon.$$

The set

$$\mathrm{LIM}^r x_i := \{x_* \in \mathbb{R}^n : x_i \xrightarrow{r} x_*\}$$

is called the r-limit set of the sequence $x = (x_i)$.

A sequence $x = (x_i)$ is said to be r-convergent if $\mathtt{LIM}^r x \neq \emptyset$. In this case, r is called the convergence degree of the sequence $x = (x_i)$. For $r = 0$, we get the ordinary convergence.

The sequence (x_n) is said to be a rough Cauchy sequence satisfying

$$\forall \varepsilon > 0, \exists n_\varepsilon : m, n \geq n_\varepsilon \Rightarrow \|x_m - x_n\| < \rho + \varepsilon$$

for $\rho > 0$. ρ is roughness degree of (x_n). Shortly (x_n) is called a rough Cauchy sequence. ρ is also a Cauchy degree of (x_n).

Lemma 1.1. [19] *Let (x_i) be r-convergent, i.e., $\mathrm{LIM}^r x_i \neq \emptyset$. Then, (x_i) is a ρ-Cauchy sequence for every $\rho \geq 2r$. This bound for the Cauchy degree cannot be generally decreased.*

2. MAIN RESULTS

In this work, we introduced the notions of rough convergence, rough Cauchy sequence and the set of rough limit points of a sequence and obtained rough convergence criteria associated with this set in 2-normed space. Later, we proved that this set is closed and convex. Finally, we examined the relations between rough convergence and rough Cauchy sequence in 2-normed space.

Definition 2.1. *Let (x_n) be a sequence in $(X, \|.,.\|)$ 2-normed linear space and r be a non-negative real number. (x_n) is said to be rough convergent (r-convergent) to L denoted by $x_n \xrightarrow{\|.,.\|}_r L$ if*

$$\forall \varepsilon > 0, \exists n_\varepsilon \in \mathbb{N} : n \geq n_\varepsilon \Rightarrow \|x_n - L, z\| < r + \varepsilon \tag{2.1}$$

or equivalently, if

$$\limsup \|x_n - L, z\| \leq r,$$

for every $z \in X$.

If (2.1) holds L is an r-limit point of (x_n), which is usually no more unique (for $r > 0$). So, we have to consider the so-called r-limit set (or shortly r-limit) of (x_n) defined by

$$\mathrm{LIM}_2^r x_n := \{L \in X : x_n \xrightarrow{\|.,.\|}_r L\}. \tag{2.2}$$

The sequence (x_n) is said to be rough convergent if

$$\mathrm{LIM}_2^r x_n \neq \emptyset.$$

In this case, r is called a convergence degree of (x_n). For $r = 0$ we have the classical convergence in 2-normed space again. But our proper interest is case $r > 0$. There are several reasons for this interest. For instance, since an orginally convergent sequence (y_n) (with $y_n \to L$) in 2-normed space often cannot be determined (i.e.,

measured or calculated) exactly, one has to do with an approximated sequence (x_n) satisfying
$$\|x_n - y_n, z\| \leq r$$
for all n and for every $z \in X$, where $r > 0$ is an upper bound of approximation error. Then, (x_n) is no more convergent in the classical sense, but for every $z \in X$,
$$\begin{aligned} \|x_n - L, z\| &\leq \|x_n - y_n, z\| + \|y_n - L, z\| \\ &\leq r + \|y_n - L, z\| \end{aligned}$$
implies that is r-convergent in the sense of (2.1).

Example 2.1. *The sequence $x = (x_n) = ((-1)^n, (-1)^n)$ is not convergent in 2-normed space $(X, \|.,.\|)$ but it is rough convergent to $L = (0,0)$ for every $z \in X$. It is clear that*
$$LIM_2^r x_n = \begin{cases} \emptyset & , \quad if \quad r < 1 \\ [(-r,-r),(r,r)] & , \quad otherwise. \end{cases}$$

Sometimes we are interested in the set of r-limit points lying in a given subset $D \subset X$, which is called r-limit in D and denoted by
$$\mathrm{LIM}_2^{D,r} x_n := \{L \in D : x_n \xrightarrow{\|.,.\|}_r L\}. \tag{2.3}$$
It is clearly
$$\mathrm{LIM}_2^{X,r} x_n = \mathrm{LIM}_2^r x_n \quad and \quad \mathrm{LIM}_2^{D,r} x_n = D \cap \mathrm{LIM}_2^r x_n.$$

First, let us transform some properties of classical convergence to rough convergence. It is well known if a sequence converges then its limit is unique. This property is nor maintained for rough convergence with roughness degree. $r > 0$, but only has the following analogy.

Theorem 2.1. *Let $(X, \|.,.\|)$ be a 2-normed space and consider a sequence $x = (x_n) \in X$. We have $diam(\mathrm{LIM}_2^r x_n) \leq 2r$. In general, $diam(\mathrm{LIM}_2^r x_n)$ has no smaller bound.*

Proof 2.1. *We have to show that*
$$diam(\mathrm{LIM}_2^r x_n) = sup\left\{\|y - t, z\| : y, t \in \mathrm{LIM}_2^r x_n \leq 2r\right\}, \tag{2.4}$$
where $(X, \|.,.\|)$ is a 2-normed space and for every $z \in X$. Assume the contrary that
$$diam(\mathrm{LIM}_2^r x_n) > 2r$$
then, there exist $y, t \in \mathrm{LIM}_2^r x_n$ satisfying
$$d := \|y - t, z\| > 2r,$$
for every $z \in X$. For an arbitrary $\varepsilon \in (0, d/2 - r)$, it follows from (2.1) and (2.2) that there is an $n_\varepsilon \in \mathbb{N}$ such that for $n \geq n_\varepsilon$,
$$\|x_n - y, z\| < r + \varepsilon \quad and \quad \|x_n - t, z\| < r + \varepsilon,$$
for every $z \in X$. This implies
$$\begin{aligned} \|y - t, z\| &\leq \|x_n - y, z\| + \|x_n - t, z\| \\ &< 2(r + \varepsilon) \\ &< 2r + 2(d/2 - r) \\ &= d, \end{aligned}$$

for every $z \in X$*, which conflicts with* $d = \|y - t, z\|$*. Hence, (2.4) must be true. Consider a convergent sequence* (x_n) *with* $\lim_{n\to\infty} x_n = L$*. Then, for*

$$\overline{B}_r(L) := \{y \in X : \|y - L, z\| \leq r\},$$

it follows from

$$\begin{aligned} \|x_n - y, z\| &\leq \|x_n - L, z\| + \|L - y, z\| \\ &\leq \|x_n - L, z\| + r \end{aligned}$$

for every $z \in X$ *and for* $y \in \overline{B}_r(L)$*, (2.1) and (2.2) that*

$$\mathrm{LIM}_2^r x_n = \overline{B}_r(L).$$

Since $diam(\overline{B}_r(L)) = 2r$*, this shows that in general the upper bound* $2r$ *of the diameter of an* r*-limit set cannot be decreased anymore.*

Obviously the uniqueness of limit (of classical convergence) can be regarded as a special case of latter property, because if $r = 0$ then $diam(\mathrm{LIM}_2^r x_n) = 2r = 0$, that is, $\mathrm{LIM}_2^r x_n$ is either empty or a singleton. A further property of classical concept is the boundedness of convergent sequences. Its analogy for rough convergence is:

Theorem 2.2. *Let* $(X, \|.,.\|)$ *be a 2-normed space and consider a sequence* $x = (x_n) \in X$*. The sequence* (x_n) *is bounded if and only if there exist an* $r \geq 0$ *such that* $\mathrm{LIM}_2^r x_n \neq \emptyset$*. For all* $r > 0$*, a bounded sequence* (x_n) *is always contains a subsequence* x_{n_k} *with*

$$\mathrm{LIM}_2^{(x_{n_k}),r} x_{n_k} \neq \emptyset.$$

Proof 2.2. *For every* $z \in X$ *if*

$$s := sup\{\|x_n, z\| : n \in \mathbb{N}\} < \infty$$

then, $\mathrm{LIM}_2^s x_n$ *contains the origin of* X*. On the other hand* $\mathrm{LIM}_2^r x_n \neq \emptyset$*, for some* $r \geq 0$ *then, all but finite elements* x_n *are contained in some ball with any radius greater than* r*. Therefore, the sequence* (x_n) *is bounded in* 2*-normed space* X*. As* (x_n) *is bounded sequence in* 2*-normed space* X*, it certainly contains a convergent subsequence* (x_{n_k})*. Let* L *be its limit point, then*

$$\mathrm{LIM}_2^r x_{n_k} = \overline{B}_r(L)$$

and for $r > 0$*,*

$$\mathrm{LIM}_2^{(x_{n_k}),r} x_{n_k} = \{x_{n_k} : \|L - x_{n_k}, z\| \leq r\} \neq \emptyset,$$

for every $z \in X$*.*

Note that the second part of the previous proposition is concerned with r-limit points lying in the subsequence (x_{n_k}) in 2-normed space X. It is straightforward that a sequence contained in some bounded set D always possesses a subsequence being r-convergent (for an arbitrary $r > 0$) to some point of D. Here, the closedness of D is not needed as for classical convergence. Corresponding to the property that each subsequence of a convergent sequence also converges to the same limit point, we now have the following one whose proof is rather simple.

Proposition 2.1. *Let* $(X, \|.,.\|)$ *be a 2-normed space and consider a sequence* $x = (x_n) \in X$*. If* (x'_n) *is a subsequence of* (x_n) *then,*

$$\mathrm{LIM}_2^r x_n \subseteq \mathrm{LIM}_2^r x'_n$$

in 2*-normed space* X*.*

Theorem 2.3. *Let $(X, \|.,.\|)$ be a 2-normed space and consider a sequence $x = (x_n) \in X$. For all $r \geq 0$, the r-limit set $\mathrm{LIM}_2^r x_n$ of an arbitrary sequence (x_n) is closed.*

Proof 2.3. *Let (y_m) be an arbitrary sequence in $\mathrm{LIM}_2^r x_n$ which converges to some point L. For each $\varepsilon > 0$ and every $z \in X$, by definition there are $m_{\varepsilon/2}$ and an $n_{\varepsilon/2}$ such that*

$$\|y_{m_{\varepsilon/2}} - L, z\| < \varepsilon/2 \quad and \quad \|x_n - y_{m_{\varepsilon/2}}, z\| < r + \varepsilon/2$$

whenever $n \geq n_{\varepsilon/2}$. Consequently for every $z \in X$,

$$\begin{aligned} \|x_n - L, z\| &\leq \|x_n - y_{m_{\varepsilon/2}}, z\| + \|y_{m_{\varepsilon/2}} - L, z\| \\ &< r + \varepsilon, \end{aligned}$$

if $n \geq n_{\varepsilon/2}$. That means $L \in \mathrm{LIM}_2^r x_n$ too. Hence, $\mathrm{LIM}_2^r x_n$ closed.

Theorem 2.4. *Let $(X, \|.,.\|)$ be a 2-normed space and consider a sequence $x = (x_n) \in X$. If*

$$y_0 \in \mathrm{LIM}_2^{r_0} x_n \quad and \quad y_1 \in \mathrm{LIM}_2^{r_1} x_n$$

then,

$$y_\alpha := (1-\alpha) y_0 + \alpha y_1 \in \mathrm{LIM}_2^{(1-\alpha) r_0 + \alpha r_1} x_n, \quad for \quad \alpha \in [0,1].$$

Proof 2.4. *By definition, for every $\varepsilon > 0$, $r_0, r_1 > 0$ and every $z \in X$ there exists an n_ε such that $n > n_\varepsilon$ implies*

$$\|x_n - y_o, z\| < r_0 + \varepsilon \quad and \quad \|x_n - y_1, z\| < r_1 + \varepsilon,$$

which yields also, for every $z \in X$,

$$\begin{aligned} \|x_n - y_\alpha, z\| &\leq (1-\alpha)\|x_n - y_o, z\| + \alpha \|x_n - y_1, z\| \\ &< (1-\alpha)(r_0 + \varepsilon) + \alpha(r_1 + \varepsilon) \\ &= (1-\alpha) r_0 + \alpha r_1 + \varepsilon. \end{aligned}$$

Hence, we have

$$y_\alpha \in \mathrm{LIM}_2^{(1-\alpha) r_0 + \alpha r_1} x_n.$$

Theorem 2.5. *Let $(X, \|.,.\|)$ be a 2-normed space and consider a sequence $x = (x_n) \in X$. $\mathrm{LIM}_2^r x_n$ is convex.*

Proof 2.5. *In particular, for $r = r_0 = r_1$, Theorem 2.4 yields immediately that $LIM_2^r x_n$ is convex.*

Theorem 2.6. *If $x_n \xrightarrow{\|.,.\|}_r L_1$ and $y_n \xrightarrow{\|.,.\|}_r L_2$. Then,*

(i) $(x_n + y_n) \xrightarrow{\|.,.\|}_r (L_1 + L_2)$ *and*

(ii) $c.(x_n) \xrightarrow{\|.,.\|}_r c.L_1, (c \in \mathbb{R})$.

Proof 2.6. (i) *By definition for every $z \in X$,*

$$\forall \varepsilon > 0, \exists n_\varepsilon \in \mathbb{N} : n \geq n_\varepsilon \Rightarrow \|x_n - L_1, z\| \leq r_1 + \frac{\varepsilon}{2}$$

and

$$\forall \varepsilon > 0, \exists j_\varepsilon \in \mathbb{N} : n \geq j_\varepsilon \Rightarrow \|y_n - L_2, z\| \leq r_2 + \frac{\varepsilon}{2}.$$

Let $j = max(n_\varepsilon, j_\varepsilon)$ and $r_1 + r_2 = r$. For every $n > j$ and every $z \in X$ we have

$$\begin{aligned} \|(x_n + y_n) - (L_1 + L_2), z\| &= \|x_n - L_1, z\| + \|y_n - L_2, z\| \\ &\leq r_1 + \frac{\varepsilon}{2} + r_2 + \frac{\varepsilon}{2} \\ &= r + \varepsilon \end{aligned}$$

and so

$$(x_n + y_n) \xrightarrow{\|.,.\|}_r (L_1 + L_2).$$

(ii) It is obvious for $c = 0$. Let $c \neq 0$. Since

$$x_n \xrightarrow{\|.,.\|}_r L$$

for every $\varepsilon > 0$ and every $z \in X$, $\exists n_\varepsilon \in \mathbb{N}$ such taht for every $n \geq n_\varepsilon$ we have

$$\|x_n - L, z\| \leq \frac{r + \varepsilon}{|c|}.$$

According to this, for $\forall n \geq n_\varepsilon$ and every $z \in X$, we can write

$$\begin{aligned} \|c.x_n - c.L, z\| &= |c|.|x_n - L, z\| \\ &\leq |c|.\frac{r + \varepsilon}{|c|} \\ &= r + \varepsilon. \end{aligned}$$

So,

$$(c.x_n) \xrightarrow{\|.,.\|}_r c.L.$$

Definition 2.2. *Let (x_n) be a sequence in $(X, \|.,.\|)$ 2-normed space. (x_n) is said to be a rough Cauchy sequence with roughness degree ρ, if*

$$\forall \varepsilon > 0, \exists k_\varepsilon : m, n \geq k_\varepsilon \Rightarrow \|x_m - x_n, z\| \leq \rho + \varepsilon$$

is hold for $\rho > 0$, $L \in X$ and every $z \in X$. ρ is also called a Cauchy degree of (x_n).

Proposition 2.2. *(i) Monotonicity: Assume $\rho' > \rho$. If ρ is a Cauchy degree of a given sequence (x_n) in $(X, \|.,.\|)$ 2-normed space, so ρ' is a Cauchy degree of (x_n).*

(ii) Boundedness: A sequence (x_n) is bounded if and only if there exists a $\rho \geq 0$ such that (x_n) is a ρ-Cauchy sequence in $(X, \|.,.\|)$ 2-normed space.

Theorem 2.7. *If (x_n) is rough convergent in 2-normed space $(X, \|.,.\|)$, i.e., $\mathrm{LIM}_2^r x_n \neq \emptyset$ if and only if (x_n) is a ρ-Cauchy sequence for every $\rho \geq 2r$. This bound for the Cauchy degree cannot be generally decreased.*

Proof 2.7. *Let L be any point in $\mathrm{LIM}_2^r x_n$ Then, for all $\varepsilon > 0$, there exists an $k_\varepsilon \in \mathbb{N}$ such that $m, n \geq k_\varepsilon$ implies*

$$\|x_m - L, z\| \leq r + \frac{\varepsilon}{2} \ \text{ and } \ \|x_n - L, z\| \leq r + \frac{\varepsilon}{2},$$

for every $z \in X$. Therefore, $m, n \geq k_\varepsilon$, we have

$$\begin{aligned} \|x_m - x_n, z\| &= \|x_m - L + L - x_n, z\| \\ &\leq \|x_m - L, z\| + \|L - x_n, z\| \\ &\leq r + \frac{\varepsilon}{2} + r + \frac{\varepsilon}{2} \\ &= 2r + \varepsilon \end{aligned}$$

for every $z \in X$. Hence, (x_n) is a ρ-Cauchy sequence for $\rho \geq 2r$. By proposition 2.2, every $\rho \geq 2r$ is also a Cauchy degree of (x_n).

Let (x_n) be a rough Cauchy sequence in 2-normed space $(X, \|.,.\|)$. Since (x_n) be a rough Cauchy sequence, then it is bounded and consequently it is rough convergent for $\rho > 0$ and for every $z \in X$. It is clear that this bound $2r$ can not be generally decreased by Lemma 1.1.

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References

[1] M. Arslan, E. Dündar, *$\mathcal{I}$-Convergence and $\mathcal{I}$-Cauchy Sequence of Functions In 2-Normed Spaces,* Konuralp Journal of Mathematics, **6**(1) (2018) 57–62.
[2] M. Arslan, E. Dündar, *On $\mathcal{I}$-Convergence of sequences of functions in 2-normed spaces,* Southeast Asian Bulletin of Mathematics, **42** (2018), 491–502.
[3] S. Aytar, *Rough statistical convergence*, Numer. Funct. Anal. and Optimiz, **29**(3-4): (2008) 291–303.
[4] S. Aytar, *The rough limit set and the core of a real requence*, Numer. Funct. Anal. and Optimiz, **29**(3-4): (2008) 283–290.
[5] H. Çakalli and S. Ersan, *New types of continuity in 2-normed spaces,* Filomat, **30**(3) (2016) 525–532.
[6] E. Dündar, C. Çakan, *Rough $\mathcal{I}$-convergence*, Gulf Journal of Mathematics, **2**(1) (2014) 45–51.
[7] E. Dündar, C. Çakan, *Rough convergence of double sequences*, Demonstratio Mathematica **47**(3) (2014) 638–651.
[8] S. Gähler, *2-metrische Räume und ihre topologische struktur*, Math. Nachr. **26** (1963) 115–148.
[9] S. Gähler, *2-normed spaces*, Math. Nachr. **28** (1964) 1–43.
[10] H. Gunawan, M. Mashadi, *On n-normed spaces*, Int. J. Math. Math. Sci. **27**(10) (2001) 631–639.
[11] H. Gunawan, M. Mashadi, *On finite dimensional 2-normed spaces*, Soochow J. Math. **27**(3) (2001) 321–329.
[12] M. Gürdal, S. Pehlivan, *The statistical convergence in 2-Banach spaces*, Thai J. Math. **2**(1) (2004) 107–113.
[13] M. Gürdal, S. Pehlivan, *Statistical convergence in 2-normed spaces*, Southeast Asian Bull. Math. **33** (2009) 257–264.
[14] M. Gürdal, I. Açık, *On $\mathcal{I}$-Cauchy sequences in 2-normed spaces*, Math. Inequal. Appl. **11**(2) (2008) 349–354.
[15] M. Gürdal, *On ideal convergent sequences in 2-normed spaces*, Thai J. Math. **4**(1) (2006) 85–91.
[16] M. Mursaleen, A. Alotaibi, *On $\mathcal{I}$-convergence in random 2-normed spaces*, Math. Slovaca **61**(6) (2011) 933–940.
[17] H. X. Phu, *Rough convergence in normed linear spaces*, Numer. Funct. Anal. and Optimiz, **22** (2001) 199–222.
[18] H. X. Phu, *Rough continuity of linear operators*, Numer. Funct. Anal. and Optimiz, **23** (2002), 139–146.
[19] H. X. Phu, *Rough convergence in infinite dimensional normed spaces*, Numer. Funct. Anal. and Optimiz, **24** (2003) 285–301.
[20] S. Sarabadan, S. Talebi, *Statistical convergence and ideal convergence of sequences of functions in 2-normed spaces*, Int. J. Math. Math. Sci. **2011** (2011) 10 pages. doi:10.1155/2011/517841.
[21] E. Savaş, M. Gürdal, *Ideal Convergent Function Sequences in Random 2-Normed Spaces,* Filomat, **30**(3) (2016) 557–567.
[22] I.J. Schoenberg, *The integrability of certain functions and related summability methods*, Amer. Math. Monthly, **66** (1959) 361–375.
[23] A. Sharma, K. Kumar, *Statistical convergence in probabilistic 2-normed spaces,* Mathematical Sciences, **2**(4) (2008) 373–390.

[24] A. Şahiner, M. Gürdal, S. Saltan, H. Gunawan, *Ideal convergence in 2-normed spaces*, Taiwanese J. Math. **11**(5) (2007) 1477–1484.
[25] S. Yegül, E. Dündar, *On Statistical Convergence of Sequences of Functions In 2-Normed Spaces*, Journal of Classical Analysis, **10**(1) (2017) 49–57.

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